XMLECTURE

14 ELECTROMAGNETISM

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14.1 Magnetic Field

The SI unit for the strength of magnetic field (aka magnetic flux density) is the tesla (symbol T). Check out the table below for strength of the magnetic field produced by different types of magnets.

Source	Magnetic Field/T
Microwave oven at 30 cm	10^{-6}
Earth's Magnetic Field	10^{-5}
Fridge magnet	10^{-3}
Neodymium magnet	10^{0}
Core of 60Hz Transformer	10^{0}
MRI	10^{0}
Superconducting electromagnet	10^{1}

For a long time, we thought that magnetism and electricity are two unrelated fields. Until it was demonstrated that an electric current produces a magnetic field.

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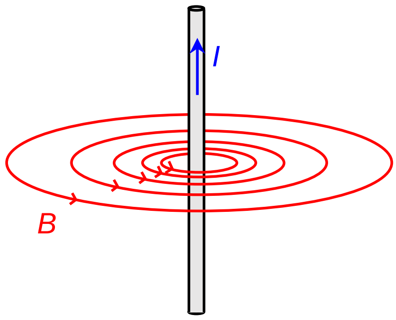
14.1.1 Long Straight Wire

How do we demonstrate that a current produces a magnetic field? One way is to arrange many tiny compasses around a long straight wire. When a current is passed through the wire, the needles of the compasses (which used to be all pointing towards the North) will rotate to reveal the direction of the magnetic field produced by the current in the wire. The magnetic field pattern turns out to be concentric circles centred about the wire!



Our right hand comes in very handy in relating the directions of the (conventional) current *I* to the direction of the magnetic field *B*. It is called the right hand grip rule (RHGR). Point your thumb in the direction of *I*, and your fingers will tell you the direction of *B*.

The strength of the magnetic field[1] at a distance *r* away from the wire is given by the formula

$$B = \frac{\mu_o I}{2\pi r}$$

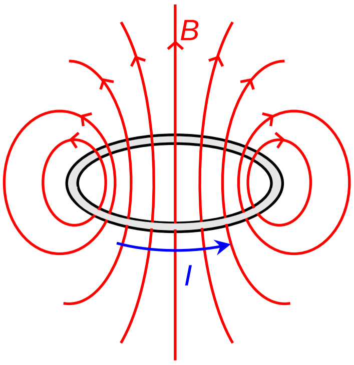
Notes:

- The constant $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1} is called the permeability of free space.
- In the H2 syllabus, *B* is more often known as the magnetic flux density.
- *B* decreases with distance from the wire. As such, the field pattern is drawn with the circles more and more spaced out as we go further and further away from the wire.

[1] I have to avoid saying "magnetic field strength" because "magnetic field strength" (not in the H2 syllabus) is actually another related but distinct quantity $H = \frac{B}{\mu_o}$.

14.1.2 Flat Circular Coil

If we fold a long straight wire into a flat circular coil, we can obtain a magnetic field at the centre of the coil that is stronger than what the long straight wire would have given us (at the distance of one radius). If we imagine the magnetic field of the straight wire folding along with the wire, it kind of makes sense that the magnetic field of the flat coil is as shown in the diagram below.



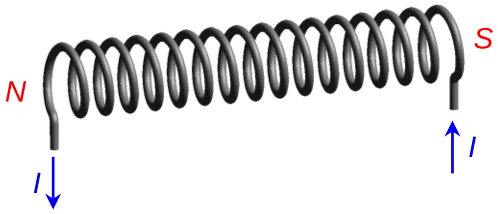
Again, the RHGR can be used to relate the direction of the (conventional) current *I* in the coil, and the direction of the magnetic field in the area enclosed by the coil. Wrap your fingers in the same direction as *I*, and your thumb will be pointing in the direction of *B*.[2]

Instead of just one turn, we can increase the magnetic field further by have more turns *N*. Assuming that the coil remains flat (negligible thickness compared to the radius *r* of the coil), the magnetic field at the centre of the flat coil of radius *r* is given by the formula

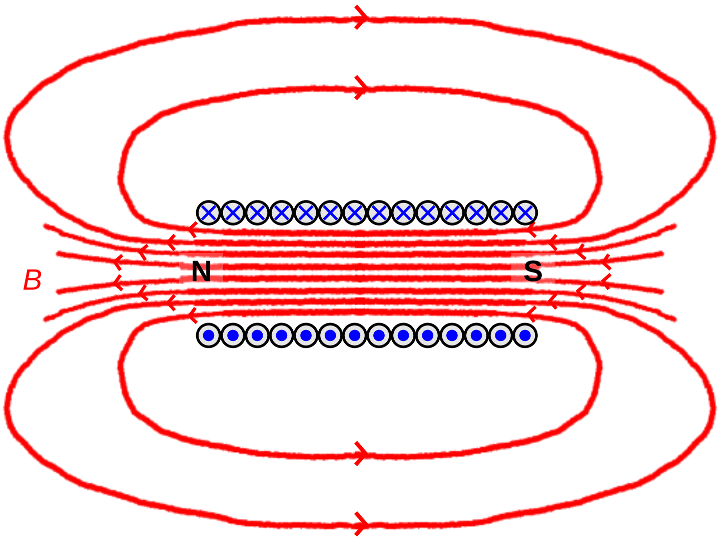
$$B = \frac{\mu_0 N I}{2r}$$

[2] So there is a RHGR for the straight wire, and a RHGR for the circular coil. Note that the roles of the thumb and fingers are switched between them. Having said that, many people apply the RHGR *along the wire* of the circular coil. That works too.

14.1.3 Long Solenoid



If we want a stronger field over a larger region of space, we can keep adding more turns (to a flat coil) until a solenoid is formed.



If the turns are wound closely enough, and the solenoid long enough, the resultant field inside the solenoid is actually uniform (reminiscent of the uniform electric field produced by large parallel plates). The RHGR can used the same way it is used for the flat coil.

The strength of the magnetic field inside the solenoid is given by the formula

$$B = \mu_o nI$$

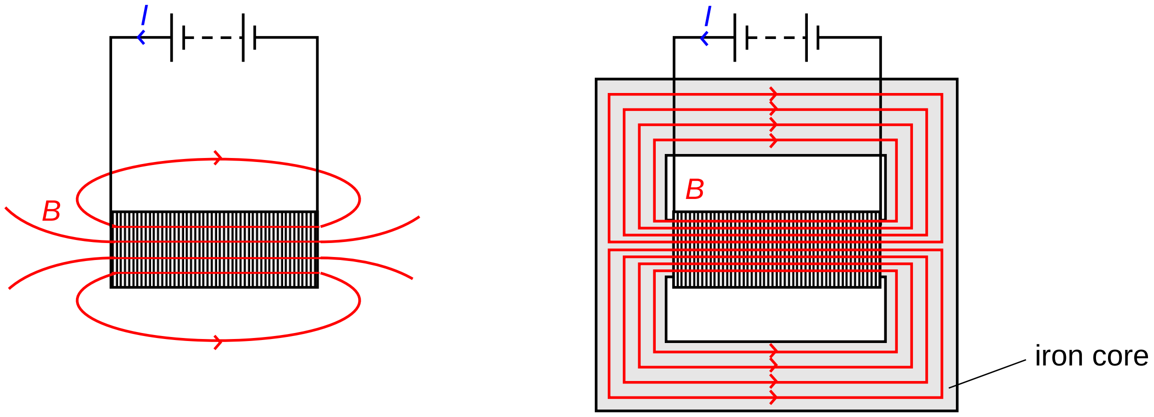
Notes:

- *n* is number of turns per unit length. e.g. 200 turns per cm.
- The field pattern of a solenoid resembles that of a bar magnet. The field lines go from the North pole to the South pole outside the solenoid, but South to North inside the solenoid.
- The density of the field lines is an indication of the strength of the magnetic field. From the field pattern, you can tell that the field is strongest (and uniform) inside the solenoid. Outside the solenoid, the strongest field is found at the poles.[3]

- [3] In fact, you can use simple logic to deduce that the strength right at the end of the long solenoid is exactly half the strength at the middle of the long solenoid.

14.1.3.1 Ferromagnetism

To increase the strength of the magnetic field further we have one more trick up our sleeve. We can wrap the turns around a piece of iron (instead of empty space).

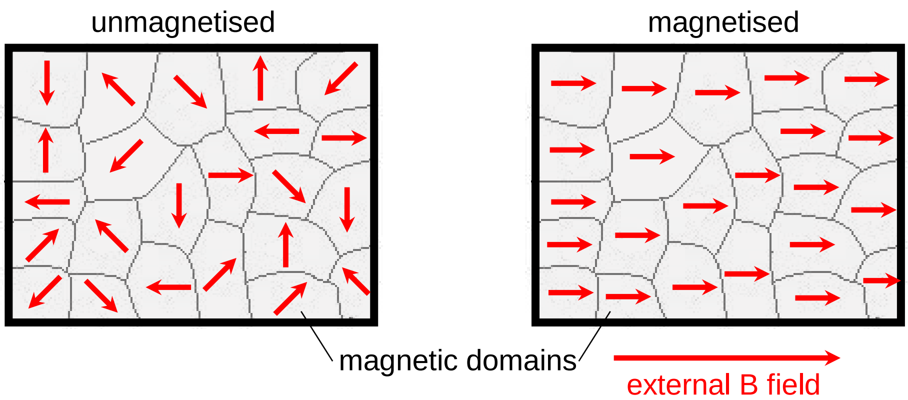


air core

iron core

The iron will be magnetized by the solenoid's magnetic field, and add its magnetic field to that of the solenoid's. Reinforced by the iron core's magnetic field, the resultant magnetic field is dramatically increased.

So how does iron (and permanent magnets) produce a magnetic field? It turns that each iron atom is a tiny magnet (something to do with this thing called electron spin). And a region where atoms are magnetized in the same direction is called a magnetic domain. An unmagnetized piece of iron is basically segmented into a large number of individual magnetic domains which are randomly oriented, so the resultant field is negligible. In the presence of an external magnetic field (such as that of the solenoid), these magnetic domains will align themselves in the direction of the external magnetic field. When aligned in the same direction, the resultant magnetic field of all these magnetic domains is substantial.



Materials which can be magnetized in this manner are said to be ferromagnetic. There are not many ferromagnetic materials. The common ones are iron, nickel, cobalt and their alloys, and some compounds of rare earth metals (e.g. neodymium).

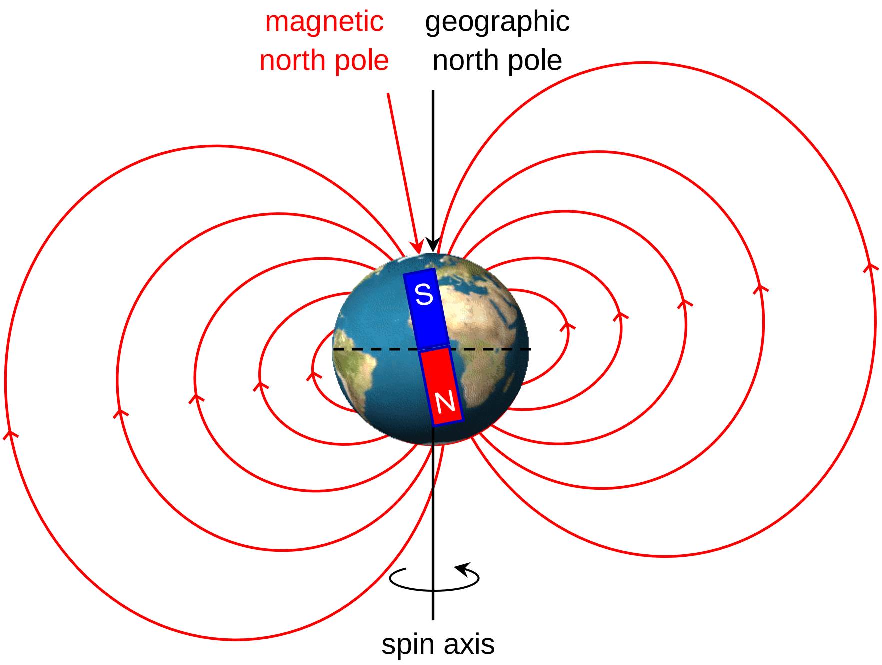
Magnetic permeability is a measure of how susceptible a substance is to the formation of a magnetic field. Compared to vacuum and most substances, ferromagnetic materials have significantly higher permeability. Check out the table below.

Material	Permeability μ / H m^{-1}	Relative permeability $\frac{\mu}{\mu_0}$
Vacuum	$\mu_0 = 4\pi \times 10^{-7}$	1
Carbon Steel	1.26×10^{-4}	100
Iron (99.8% pure)	6.3×10^{-3}	5,000
Cobalt-iron	2.3×10^{-2}	18,000
Permalloy (80% nickel, 20% iron)	1.25×10^{-1}	100,000
Most other materials, e.g. air, water, copper	$\sim 1.26 \times 10^{-6}$	~1

Permeability is another way to "explain" why the magnetic field of a solenoid with an iron core is stronger (compared to an air core one). In fact, the strength of an iron-core solenoid can be calculated using the same solenoid formula, but replacing μ_0 with μ of iron.

$$B = \mu n I$$

14.1.4 Earth's Magnetic Field



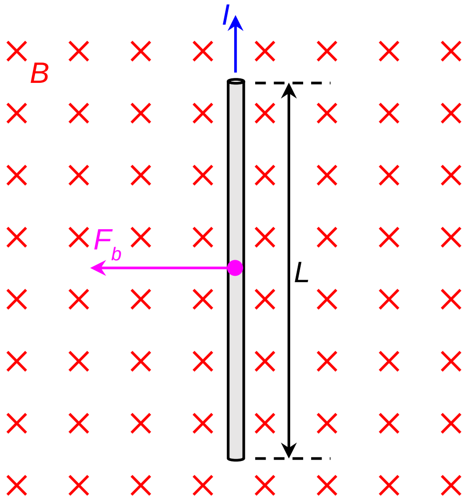
Surrounding the Earth's solid metal core is a layer of hot, liquid metal. Due to the Earth's rotation perhaps (I honestly don't know), this liquid metal is flowing. Scientists believe that the electric currents produced by this flowing liquid metal is the origin of the Earth's magnetic field. The resulting field resembles that of a bar magnet. So the earth is like a gigantic (but not so strong) "bar magnet".

As illustrated, this "bar magnet" may look flipped to many people because its north and south poles are near the geographic south and north poles respectively. The diagram is not wrong. The "bar magnet" must be oriented this way or else the Earth's magnetic field would not be directed northward.

Note that the "bar magnet" is not completely aligned with the Earth's axis of rotation (actually it is not even centred with the Earth). Currently, it is off by about 11°. This misalignment means that the Earth's magnetic field is neither horizontal (to the ground) nor northward at most places on Earth's surface. As a result, a compass needle does not point exactly towards geographic north. This deviation (called the angle of declination) can be as much as 20° at some places. If a compass is held in a vertical orientation, its needle will reveal a dip angle (also called the magnetic inclination). At the magnetic north pole, the dip angle is 90°, with the needle pointing directly into the ground!

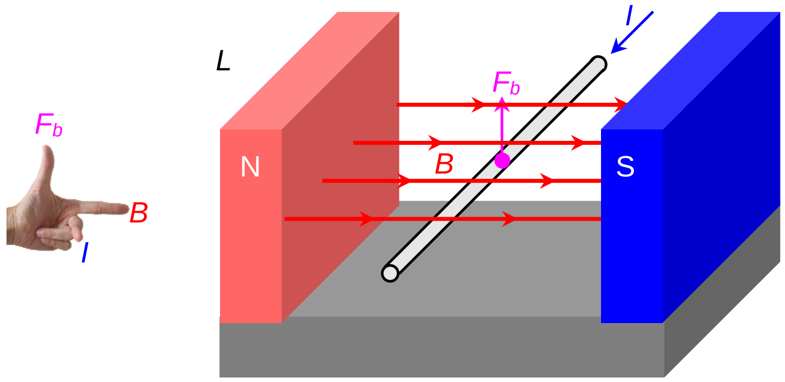
14.2 Magnetic Force on a Current Carrying Conductor

A magnet can exert a magnetic force on another magnet. Since a current carrying conductor is kind of like a magnet (since it has a magnetic field), it is not surprising that a magnetic field can exert a magnetic force on a current carrying conductor.

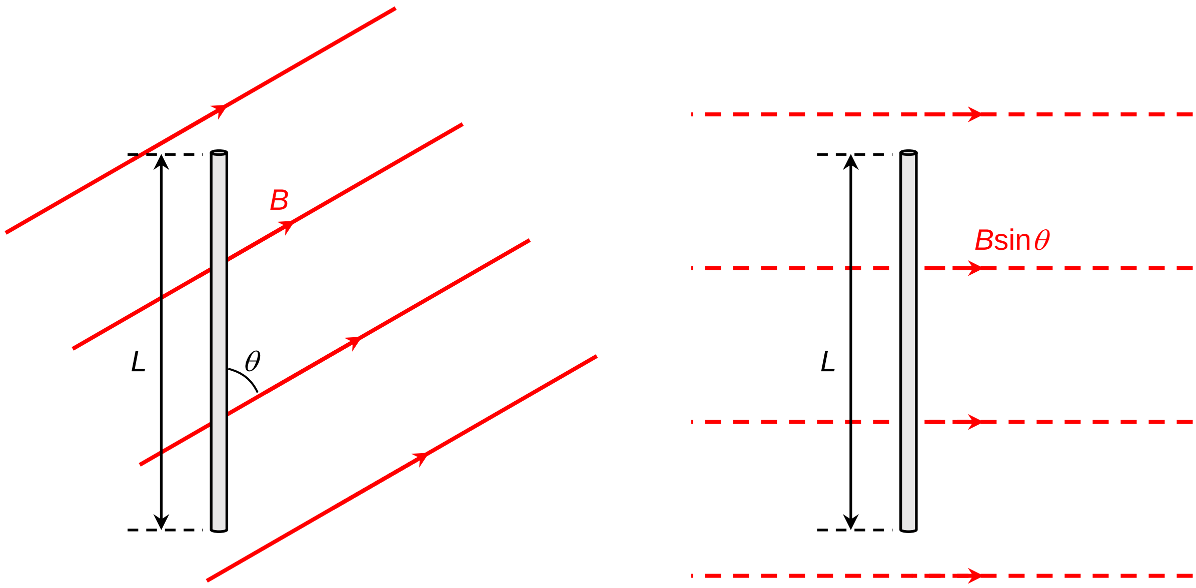


Let's say a straight wire carrying a current *I* is placed perpendicular to a uniform magnetic field *B*. If the length of the wire immersed in *B* is *L*, the magnitude of the magnetic force F_b exerted on the wire is given by the formula

$$F_b = BIL$$



The direction of the force is even more interesting. F_b is in the direction of neither *B* nor *I*. In fact, F_b is always in a third direction, the direction that is perpendicular to both *B* and *I*. Physics students all over the world use the Fleming's Left Hand Rule (FLHR) to relate the directions between F_b, *B* and *I*. The index finger is *B*, the middle finger is *I* and the thumb is F_b.



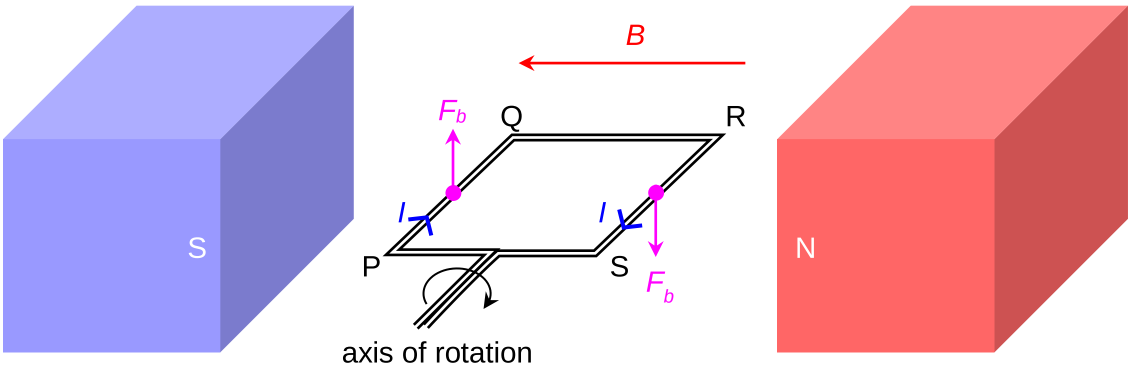
Now, what if B and I are not perpendicular to each other? Well, we know that if the wire is placed parallel to the magnetic field, zero magnetic force is exerted on the wire. This tells us that only the component of B perpendicular to the current, $B_\perp$, exerts a force on the current. So if B and I are misaligned by angle θ, then

$$\begin{aligned} F_b &= B_\perp IL \\ &= (B\sin\theta)IL \\ &= BIL\sin\theta \end{aligned}$$

14.2.1 Electric Motor

The discovery that electricity and magnetism when put together can produce a force leads directly to the invention of the electric motor.

Below is the schematic of a very basic electric motor: a rectangular coil (length $L = PQ$ and width $W = QR$) of N turns, carrying current I, rotating in a uniform magnetic field B.

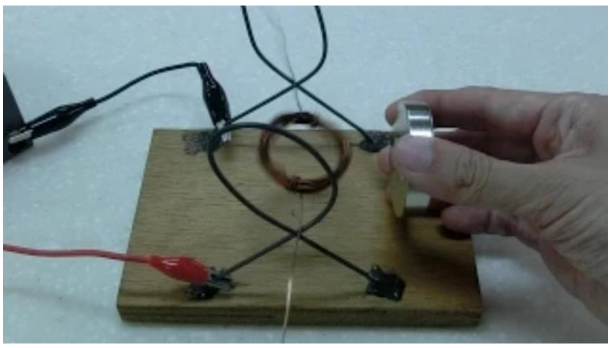


Firstly, we note that the currents I in PQ and RS are perpendicular to B, resulting in a magnetic force of

$$F_b = NBIL$$

(We can ignore the magnetic forces acting on the other two sides because they do not produce any moment about the rotational axis.)

Since the current in PQ and RS are running in opposite directions, the magnetic force F_b acting on them are also in opposite directions (using the FLHR, you can verify that F_b is upward for PQ, downward for RS). This pair of F_b forms a couple, producing a clockwise torque.



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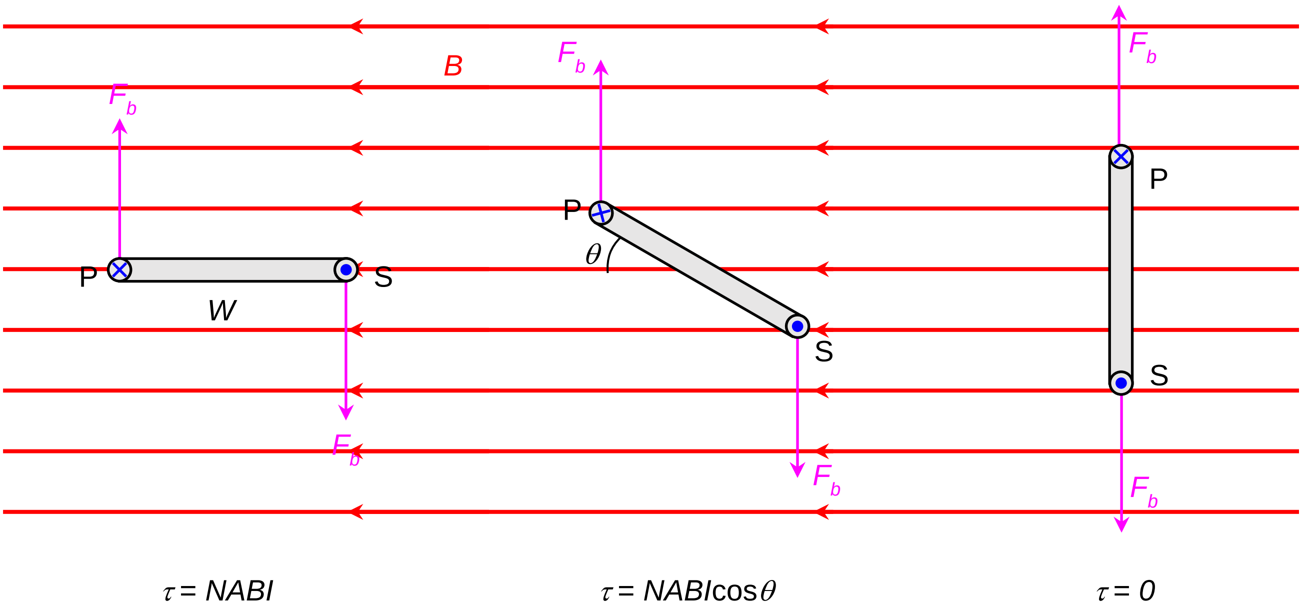
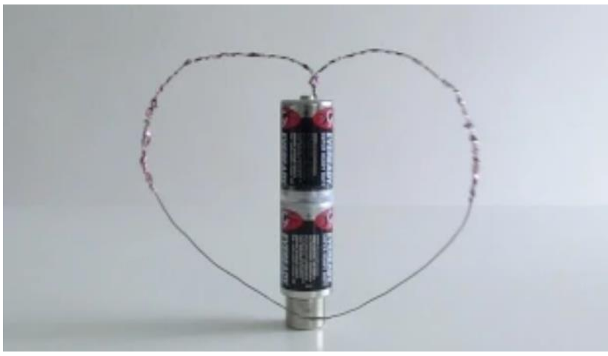


The torque of the couple τ is dependent on the perpendicular distance $W\cos\theta$. So

$$\tau = NBIL \times W\cos\theta$$
$$= NABI\cos\theta$$

where A is the area of the rectangular coil.

Do realize that the torque varies sinusoidally[4] because of the changing pivot length. The magnetic force F_b is actually constant because, look carefully, I remain perpendicular to B throughout the rotation.



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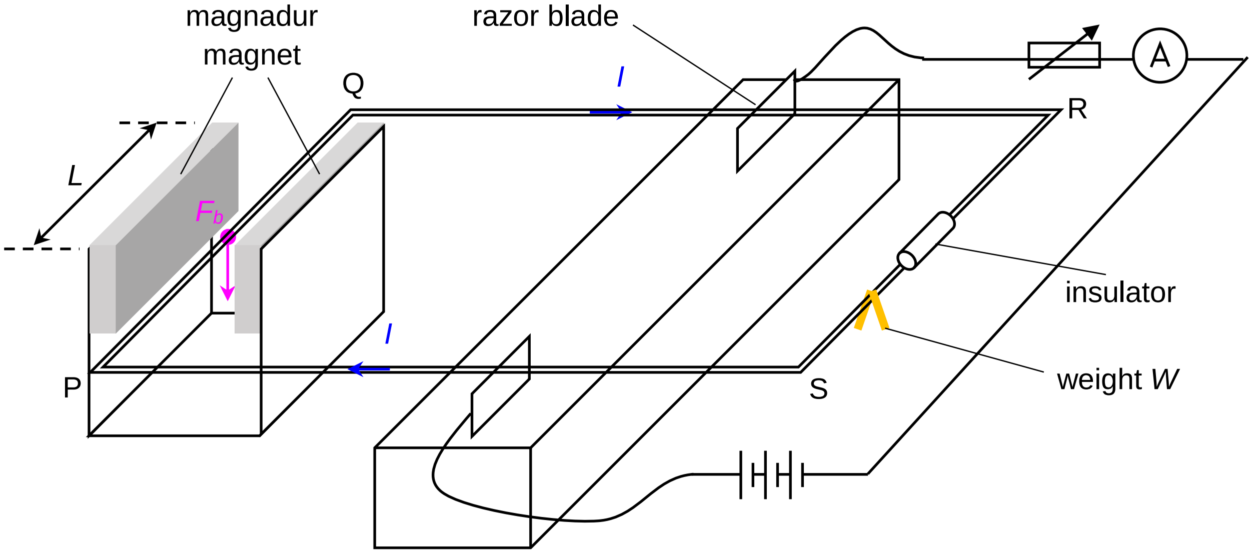
[4] It is usually desirable to have a constant torque. Practical motors employ more sophisticated magnetic fields to achieve that.

14.2.2 Current Balance

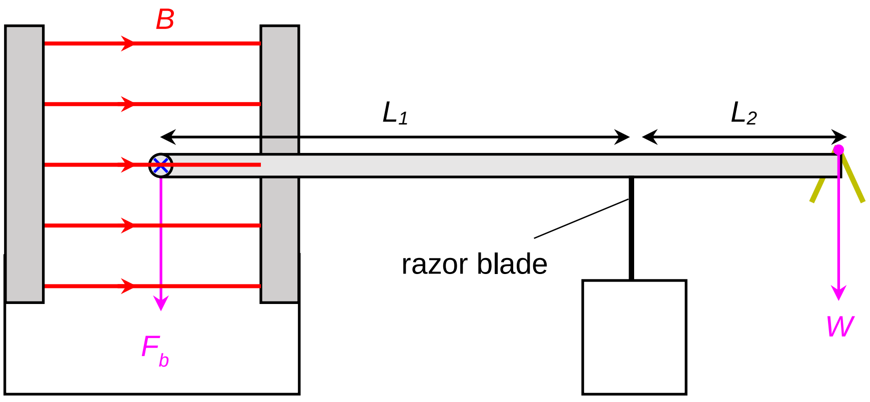
Historically, the strength of a magnetic field is obtained by measuring the force it exerts on a current carrying conductor.

$$F_b = BIL$$
$$B = \frac{F_b}{IL}$$

Basically, we are going to measure the magnetic force per unit current per unit length. The set up (see below) is called a current balance.



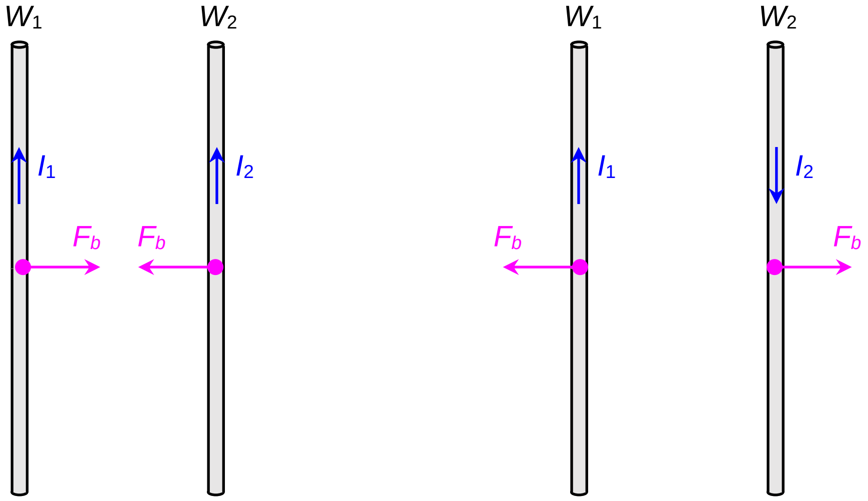
For example, to measure the strength of the magnetic field between the pair of magnadur magnets, we position side PQ of a rectangular coil perpendicularly to the field. We then balance the coil on a pair of razor blades, like a see-saw. We now (1) add a weight *W* to the RS side of the coil, and (2) pass a current through the coil. The rheostat is adjusted until the rotational equilibrium is restored.



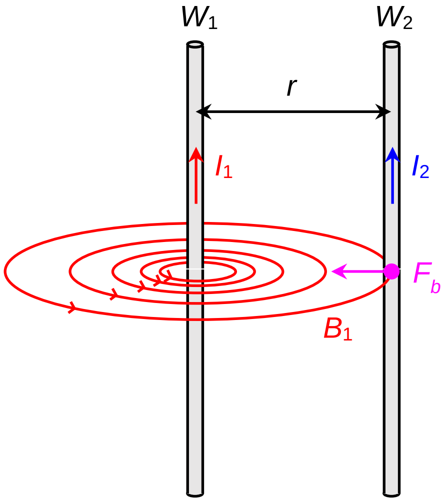
The magnitude of *B* can then be calculated by the balancing moments about the razor blade pivot:

$$F_b \times L_1 = W \times L_2$$
$$BIL \times L_1 = W \times L_2$$
$$B = \frac{W}{IL}\frac{L_2}{L_1}$$

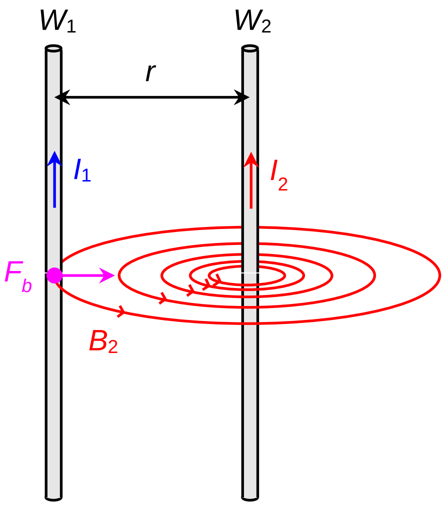
14.2.3 Forces between Two Straight Wires



For electric fields, like charges repel, and unlike charges attract. We have something similar for magnetic fields: like currents attract, and unlike currents repel.



Consider two long parallel wires W_1 and W_2 spaced r apart with currents I_1 and I_2 in the same direction. I_1 is going to produce a magnetic field of strength $B_1 = \frac{\mu_0 I_1}{2\pi r}$ at W_2. W_2 is carrying a current I_2, and sitting in B_1. It thus experiences a force per unit length of $\frac{F_b}{L} = B_1 I_2 = \frac{\mu_0 I_1}{2\pi r} I_2 = \frac{\mu_0 I_1 I_2}{2\pi r}$.



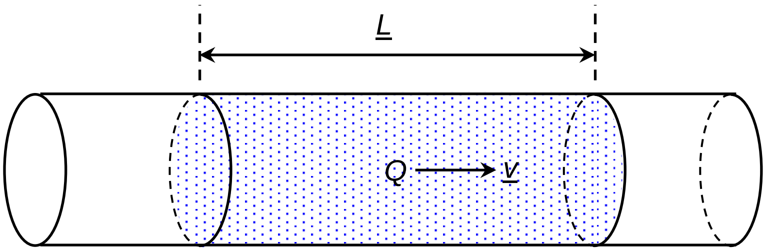
What about W_1? Similarly, W_1 is carrying I_1 sitting in B_2. So it experiences a force per unit length of $\frac{F_b}{L} = B_2 I_1 = \frac{\mu_0 I_2}{2\pi r} I_1 = \frac{\mu_0 I_1 I_2}{2\pi r}$.

Using the FLHR, one can deduce that the two wires are mutually attracting each other if they carry currents in the same direction. If W_1 and W_2 are carrying currents in opposite directions, then it is going to be a mutual repulsion.

It shouldn't surprise you that the two wires always exert an equal but opposite force on each other. They are just obeying N3L.

14.3 Magnetic Force on a Moving Charge

So a magnetic field exerts a magnetic force on a current-carrying conductor. But a current is nothing more than a flow of electric charges. Could it be that the magnetic field is actually exerting magnetic forces on the moving charges? So the magnetic force acting on the conductor is actually the summation of the magnetic forces acting on each individual electron drifting through the wire. If that's so, we should be able to derive the formula for the magnetic force acting on a moving charge, starting from the $F_b = BIL$ formula!



Let's consider a section of a copper wire of length *L*. Let the total amount of charge (carried by the free electrons) in this section be *Q*.

If *v* is the drift velocity of the electrons, then the time taken for *Q* to pass out of this section would be $L \div v$. So

$$\begin{aligned} F_b &= BIL \\ &= B\frac{Q}{t}L \\ &= B\frac{Q}{L \div v}L \\ &= BQv \end{aligned}$$

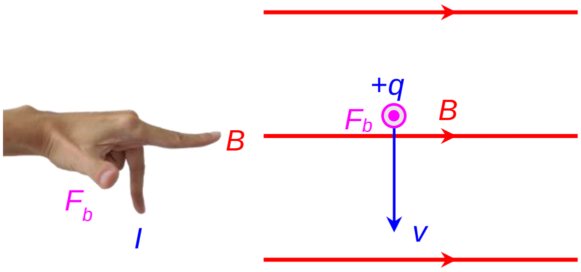
Using simple deduction, the magnetic force acting on a charged particle carrying charge *q* is given by the formula

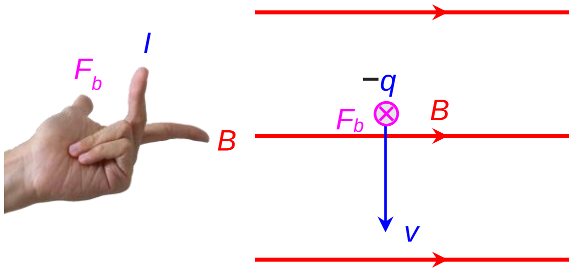
$$F_b = Bqv$$

This is of course for the case when *B* and *v* are perpendicular. If they are not, we simply replace *B* with $B_\perp$, the component of *B* perpendicular to *v*.

$$F_b = B_\perp qv$$

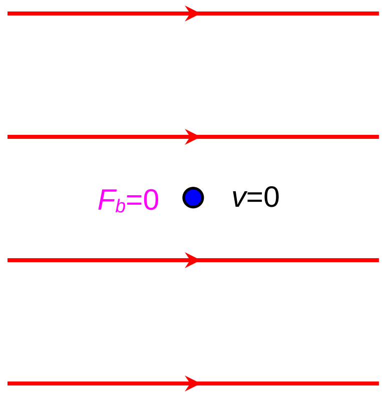
As for the direction, we can apply the FLHR for $F_b = Bqv$ the same way we apply it for $F_b = BIL$. The only thing to note is the middle finger must be pointing in the direction of conventional current, which need not be the direction of *v*. If the charge carriers are negative, the middle finger should be pointing in opposite direction to *v*.





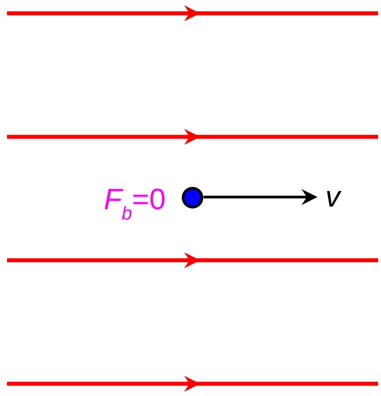
14.3.1 Charge Moving in Magnetic Field

Stationary Charge



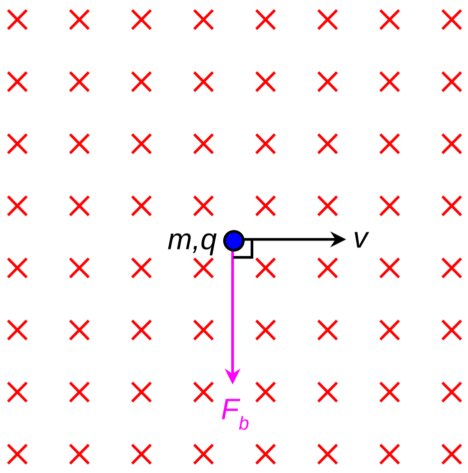
Just like a wire carrying zero current experiences no magnetic force, a stationary charge does not experience any $F_b = Bqv$. Because $v = 0$. So it remains stationary.

v parallel to *B*

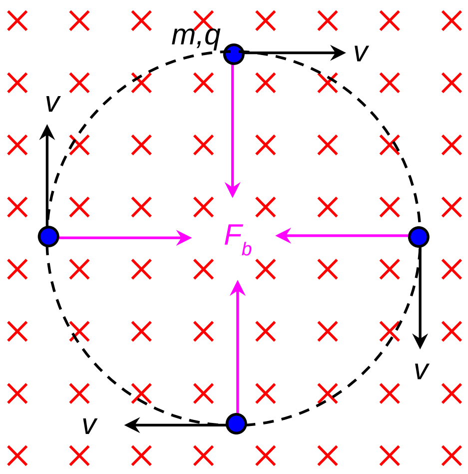


Just like a current carrying conductor placed parallel to a magnetic field experiences no magnetic force, a charge moving parallel to the direction of *B* does not experience any magnetic force $F_b = B_{\perp}qv$. Because $B_{\perp} = 0$. So it continues on its straight line motion at constant speed.

v perpendicular to B



A charge moving perpendicular to the direction of B experiences a magnetic force $F_b = B_\perp qv$. This force is guaranteed by the laws of physics to act at right angle to the velocity. If F_b is the only force acting on this charge, then the charge will be accelerating towards the centripetal direction all the time, resulting in a circular path! (This is not so for the electrons drifting in a current-carrying conductor because they are constrained by the metallic bonds to drift inside the wire)



Do realize that F_b remains perpendicular to v no matter how v rotates and turns. Charges are destined to travel in circles around magnetic field lines!

For a particle of mass m and charge q, we have

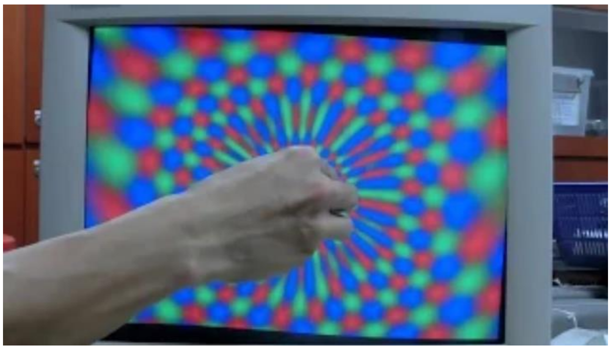
$$(F_{net} = ma)$$
$$Bqv = m\frac{v^2}{r}$$
$$r = \frac{mv}{Bq}$$

To find T, the time taken for one complete revolution, we can divide the distance by speed:

$$T = \frac{2\pi r}{v}$$
$$= (2\pi \frac{mv}{Bq}) \div v$$
$$= \frac{2\pi m}{Bq}$$

Two interesting results:

1. For the same v and B, the radius of circular motion is directly proportional to $\frac{m}{q}$, the mass to charge ratio. This relationship is the basis for mass spectroscopy.
2. The time taken for a charged particle to complete one revolution is independent of its speed. This result is exploited in the design of particle accelerators called cyclotrons.



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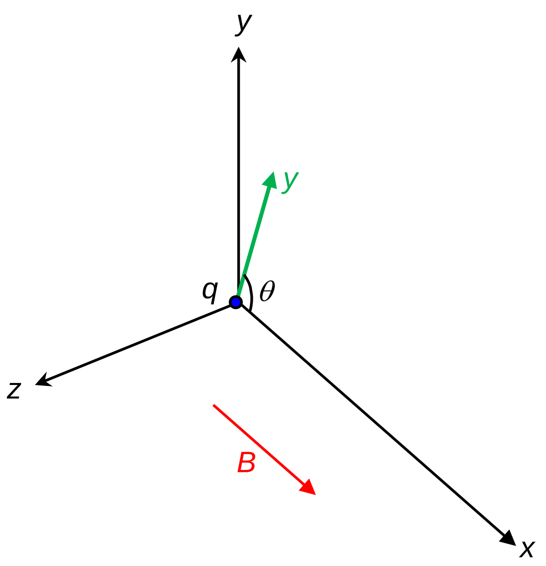
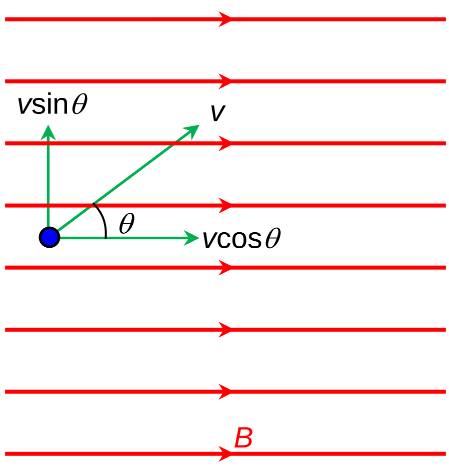
<u>*v* at angle to *B*</u>

What happens if a charged particle is moving at an angle (neither parallel nor perpendicular) to the magnetic field *B*?



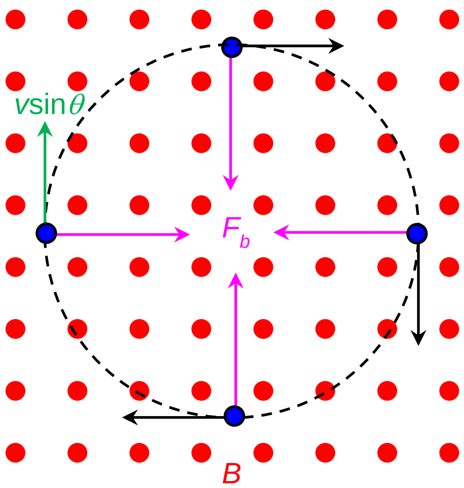
For example, in the diagram above, we have a uniform B-field directed in the +*x* direction. A positive charge *q* is moving at velocity *v* in the *x*-*y* plane, making an angle θ with the *x*-axis.



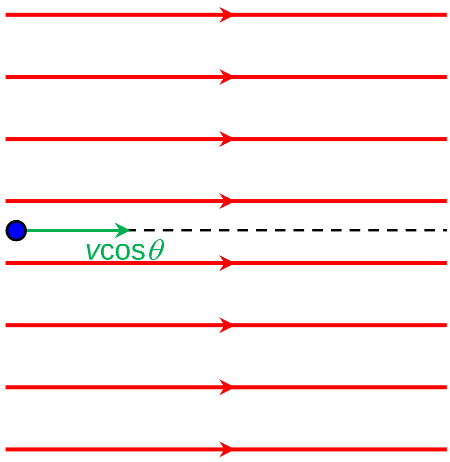
The analysis becomes rather obvious if we make an astute decision to resolve the velocity into

(1) the component parallel to *B*, $v_{\parallel} = v\cos\theta$ and

(2) the component perpendicular to *B*, $v_{\perp} = v\sin\theta$.



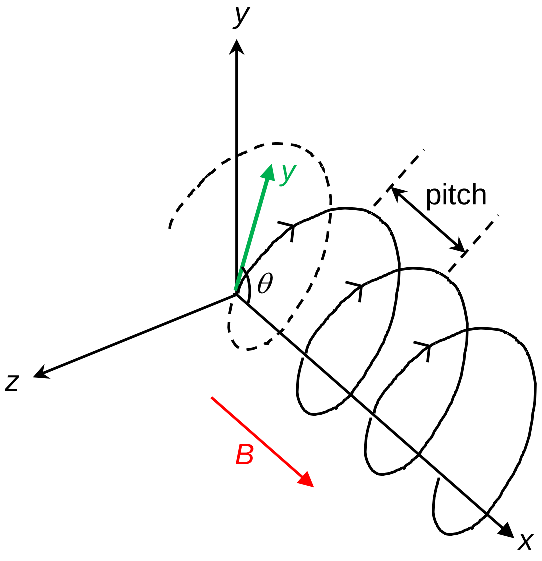
circular motion in the *y-z* plane



constant speed motion in the *x*-direction

If the charge had only velocity $v_{\parallel}$, it would have continued travelling forward (in the +x direction) at constant speed. Since $v_{\parallel}$ is parallel to *B*, there is no F_b. So $v_{\parallel}$ remains constant. (Reminiscent of the horizontal velocity in projectile motion).

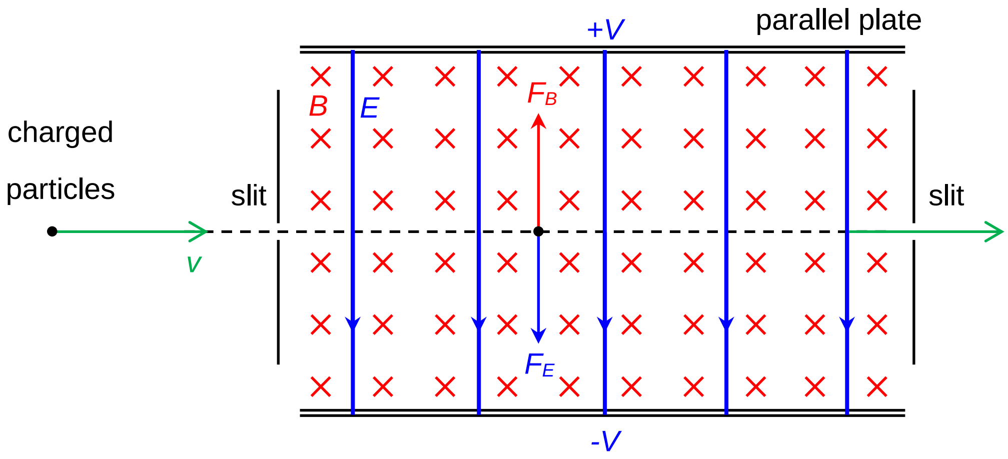
If the charge had only velocity $v_{\perp}$, its motion would have been circular motion of radius $r = \frac{mv_{\perp}}{Bq}$ in the *y-z* plane. Since $v_{\perp}$ is perpendicular to *B*, there is the centripetal magnetic force $F_b = Bqv_{\perp}$. Do realize that the centripetal acceleration is constant in magnitude. (Similar to projectile motion having a constant vertical acceleration).



Since the charge had both $v_{\parallel}$ and $v_{\perp}$, we have to superpose the two motions together. What we get is a circular motion of radius $r = \frac{mv_{\perp}}{Bq}$ that moves forward at a constant speed $v_{\parallel}$. The resultant path is a helical one.

14.3.2 Velocity Selector

In many applications such as electron microscopy and mass spectrometers, it is necessary to filter the charged particles based on their velocity. The device to do such a thing is called a velocity selector.



E-field is directed downward

B-field is directed into the page

The working principle of the velocity selector is rather straightforward (pun intended). All the ions (with all the different velocities) are made to pass through a region of uniform electric and magnetic field. The directions of the B and E fields are arranged in such a manner that each charged particle experience a magnetic force $F_b = Bqv$, and an electric force $F_e = qE$, but in opposite directions.

For the ions to pass through undeflected, the two forces must be equal in magnitude (but opposite in direction). So

$$F_b = F_e$$
$$Bqv = qE$$
$$v = \frac{E}{B}$$

It is clear now that only ions travelling at the chosen speed of $v = \frac{E}{B}$ can travel along a straight line to be collected at the exit. Ions travelling at other speeds will be deflected either upward or downward, away from the exit point and thus filtered away.

14.A Magnetic Toys

Well, these magnetic toys are definitely not required by H2 syllabus. But they are fun to watch anyway.

watch videos at xmphysics.com