XMLECTURE

15 ELECTROMAGNETIC INDUCTION

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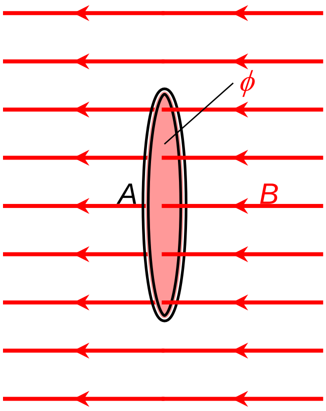
15.1 Faraday's Law

15.1.1 Magnetic Flux

Electromagnetic Induction is arguably the 2nd most mind blowing topic in the H2 syllabus. Before we start, let's get acquainted with this quantity called the magnetic flux ϕ.

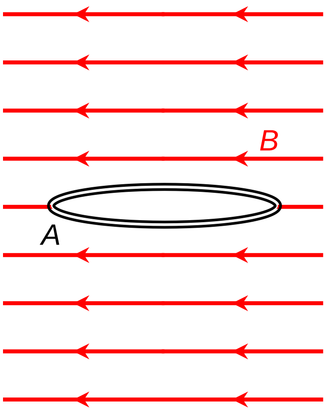


Imagine a coil with cross sectional area *A* directly facing a uniform magnetic field *B*. Does it look like *B* is passing through *A*? That magnetic thingy which is "captured" by the coil is called the magnetic flux ϕ. Quantitatively, it has the formula

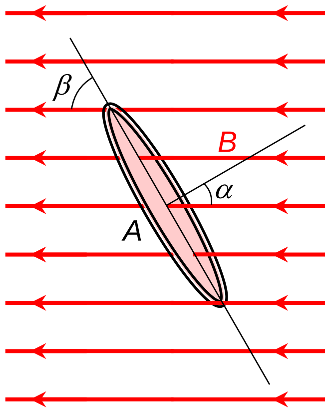
$$\phi = BA$$

The SI unit for ϕ is the weber (symbol Wb). 1 Wb is 1 T m^2.

Btw, notice that $B = \frac{\phi}{A}$. This is why, the strength of the magnetic field *B*, is also called the magnetic flux density.

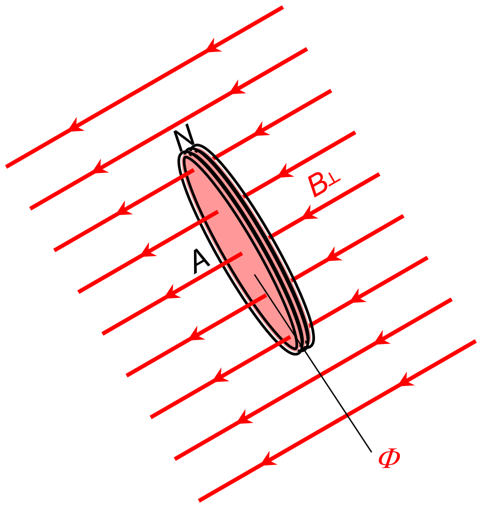


If the coil is rotated 90° so its plane is parallel to *B*, then the area *A* will not be "capturing" any *B* at all. So the magnetic flux (of the coil) is now zero.



If the axis of the coil is oriented at an angle α to *B* (so the plane of the coil makes an angle β with *B*), then the area *A* is capturing only the component of *B* perpendicular to it. So

$$\begin{aligned}\phi &= B_{\perp}A \\ &= (B\cos\alpha)A \\ &= BA\cos\alpha \text{ (or } BA\sin\beta)\end{aligned}$$



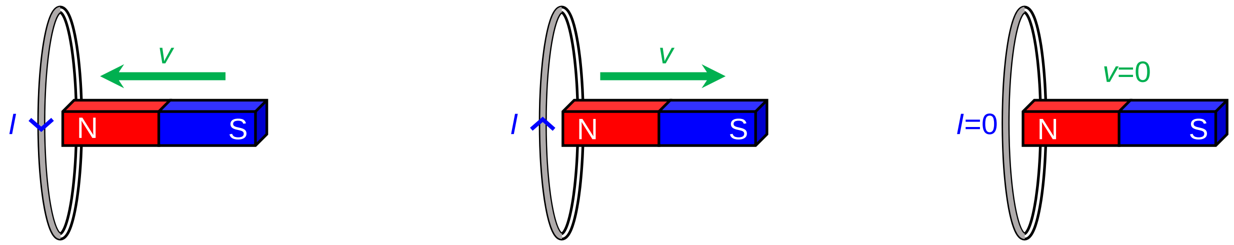
Now, if the coil has *N* turns, then each turn will be capturing a magnetic flux of $\phi = B_{\perp}A$. Logically, the coil will be capturing a total flux of $NB_{\perp}A$. We have a term specially to denote the total flux captured by a coil of many turns. It is called the magnetic flux linkage Φ.

$$\Phi = NB_{\perp}A$$

The SI unit for Φ is presumably the weber-turn (symbol Wb-turn).

15.1.2 Transformer EMF

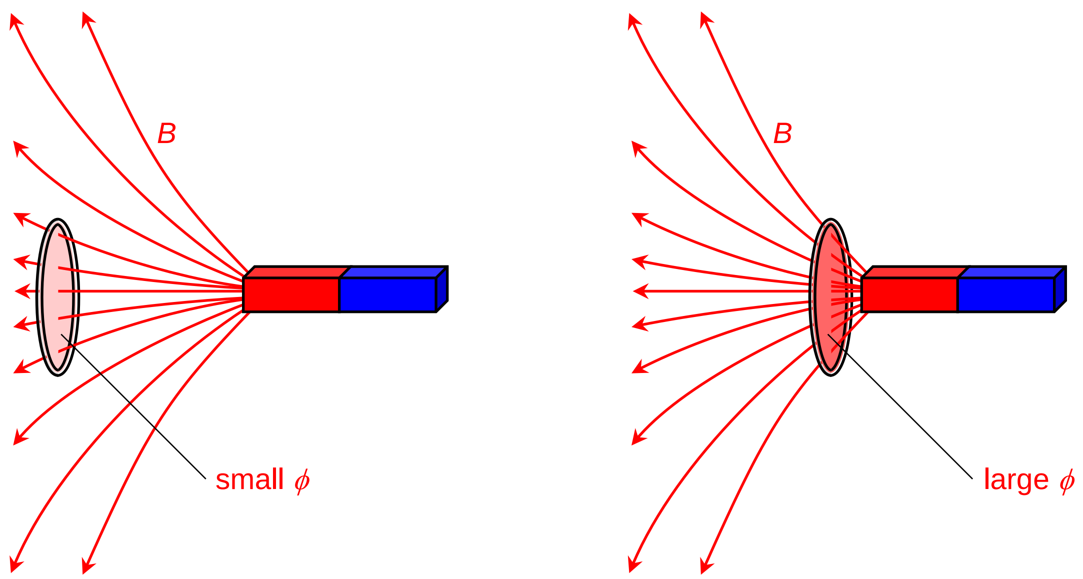
How can you produce an electric current in a copper ring, using just a magnet?



Easy. Just keep moving the magnet in and out of the ring. In the diagram above, an anticlockwise current (looking leftward) is produced when the magnet is moving towards the ring. A clockwise current (looking leftward) is produced by the magnet is moving away from the ring. But there is no current in the ring when the magnet is stationary (regardless of the position of the magnet).

It is also observed that the faster the magnet moves, the larger the current. Also, the smaller the area of the ring, the smaller the current.

So what's happening? Michael Faraday had it all figured out in 1831. He created this concept of magnetic flux linkage of a coil (introduced in the previous section). If the flux linkage is constant, nothing happens. But whenever the flux linkage changes, then an emf ε is induced in the coil. If the coil is a conducting one, then an induced current will result.



Back to the magnet and the coil. Recall that the magnetic field of a bar magnet is not a uniform one. It is stronger near the poles and weaker further away. So when the distance between the magnet and the coil changes (which occurs when the magnet is moving), so does the flux linkage of the coil. As Faraday pointed out, a changing flux linkage (of the coil) is always accompanied by an induced emf (in the coil). This induced emf is responsible for pushing the (induced) current around the coil.

It won't be physics until we have a formula to quantify our philosophy. Faraday's Law of Electromagnetic Induction states that the magnitude of the induced emf ε is directly proportional to the rate of change of magnetic flux linkage (of the coil)

$$\varepsilon = -\frac{d\Phi}{dt}$$

Why is there a negative sign? Ah, that brings us to Lenz's Law.

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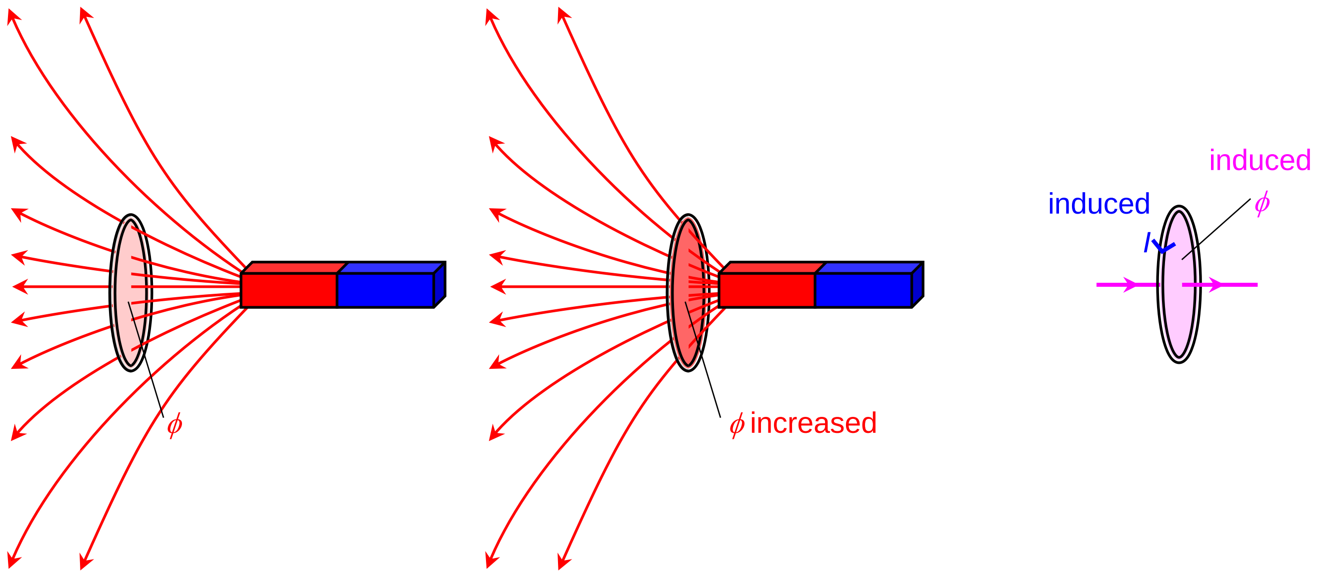
15.1.3 Lenz's Law

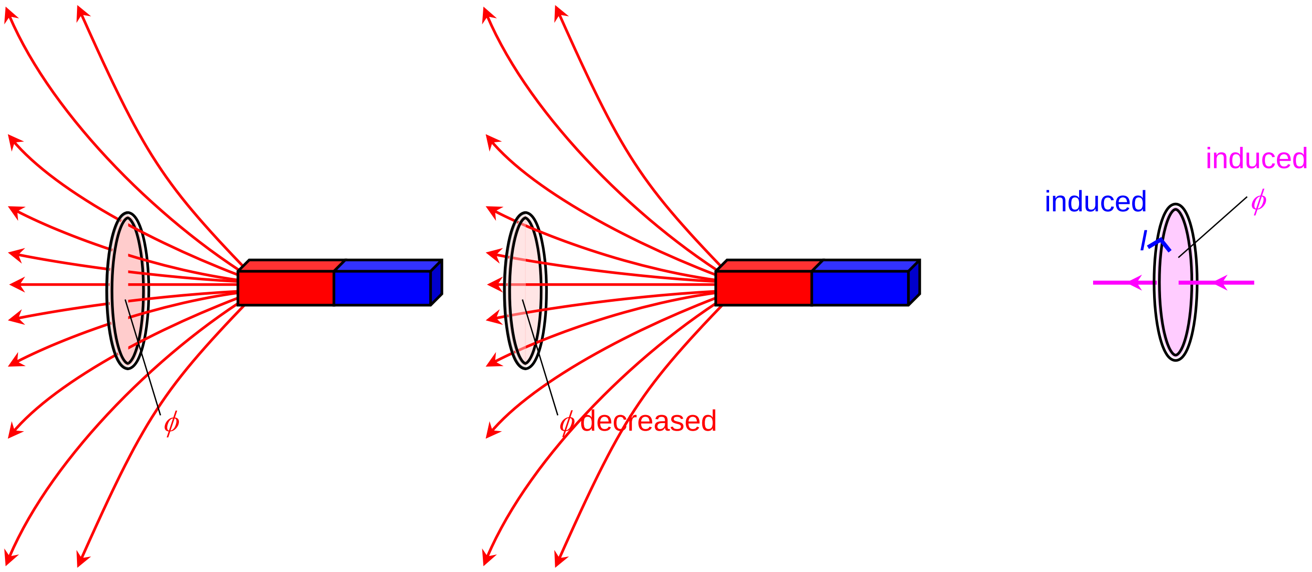
Lenz's Law bypasses the mathematics and provides an intuitive short-cut to figuring out the direction of the induced emf: the induced emf is always in the direction that opposes (or tries to oppose) the cause of the induction.



increasing flux due to approaching magnet → induced flux is in opposite direction

Take for example when emf is induced by an approaching magnet. Since the cause of the induction is an increasing leftward magnetic flux (of the coil), the coil should (try to) produce a rightward magnetic flux of its own. This requires an anticlockwise current in the coil. So we can deduce that the induced emf (and thus current) in the coil should be anticlockwise.

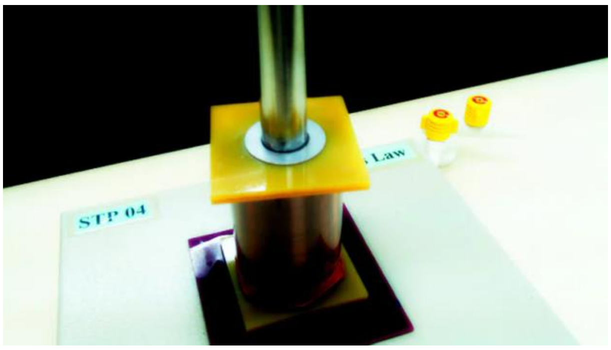
Alternatively, one may see the approaching magnet as the cause of induction. To oppose this change, the coil should (try to) repel the magnet away. This requires an anticlockwise current in the coil so that the magnetic field of the coil has its north pole facing the north pole of the approaching magnet. So we arrive at the same conclusion as before.



decreasing flux due to retreating magnet → induced flux is in the same direction

How about the case when emf is induced by a retreating magnet? Since the cause of the induction is a decreasing leftward magnetic flux (of the coil), the coil should (try to) produce a leftward magnetic flux of its own. That requires a clockwise current in the coil. So we can deduce that the induced emf (and thus current) in the coil should be clockwise.

Alternatively, one may see the magnet moving away as the cause of induction. To oppose this change, the coil should (try to) attract the magnet to keep it from leaving. This requires a clockwise current in the coil so that the magnetic field of the coil has its south pole facing the north pole of the retreating magnet. So we arrive at the same conclusion as before.



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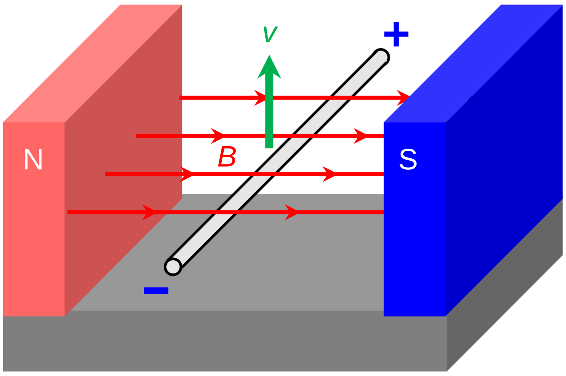
Some astute students may recognize Lenz's Law as a manifestation of the Principle of Conservation of Energy. Note that an approaching magnet produces electricity in the coil. The creation of electrical energy must be at the expense of the kinetic energy of the magnet. If the magnet is being attracted instead of being repelled (meaning the change is reinforced instead of being opposed), the magnet will be gaining KE even as it is inducing electrical energy. That's a clear violation of PCOE!

For the situation of a retreating magnet, the magnet ought to be experiencing an attractive magnetic force. Only then can the creation of electrical energy in the coil be balanced by a corresponding decrease in the KE of the magnet.

In practice, the magnet can be kept going at constant speed by an external force acting in the direction of the magnet's velocity (leftward for the approaching magnet, rightward for the retreating magnet). In this case, the creation of electrical energy is accounted for by the (positive) work done by the external force on the magnet.

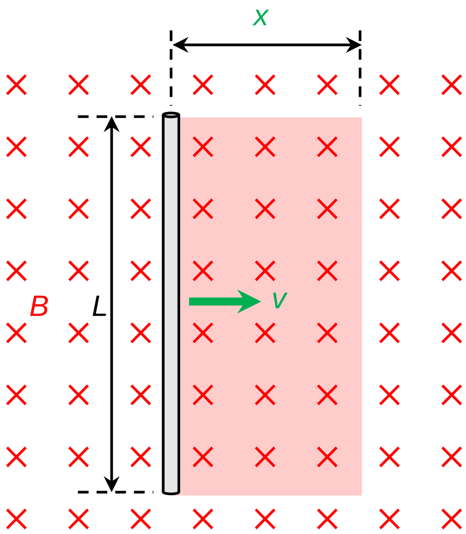
15.1.4 Motional EMF

A changing magnetic flux linkage is not the only way to induce an emf. We know as a fact that when a wire is moved across a (constant) magnetic field, an emf is induced between the two ends of the wire!



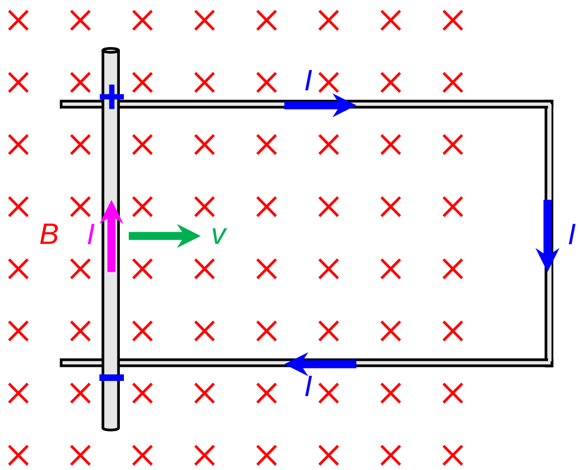
At first glance, the emf induced in a coil and a moving wire seem like two completely different phenomena. Firstly, a wire does not have any area. So it does not have any magnetic flux like a coil does. Furthermore, the wire is moving across a constant magnetic field, so there is no change to talk about. Right?

Once again, Faraday had it all figured out. A wire may not have any area, but it does sweep out an area when it is moving. So we can associate a magnetic flux to the area swept by the wire. If we interpret $\frac{d\phi}{dt}$ to be the rate at which the magnetic flux (of the swept area) is "cut" by the wire, we can kind of link these two phenomena together.



For a straight wire of length *L* moving at speed *v* (perpendicularly) across a uniform magnetic field *B*, the wire would be sweeping out a rectangular area of *Lx*, which has an associated flux of *BLx*. So the rate of flux cut by the wire is

$$
\begin{aligned}
|\varepsilon| &= \frac{d\phi}{dt} \\
&= \frac{d(BLx)}{dt} \\
&= BL\frac{dx}{dt} \\
&= BLv
\end{aligned}
$$



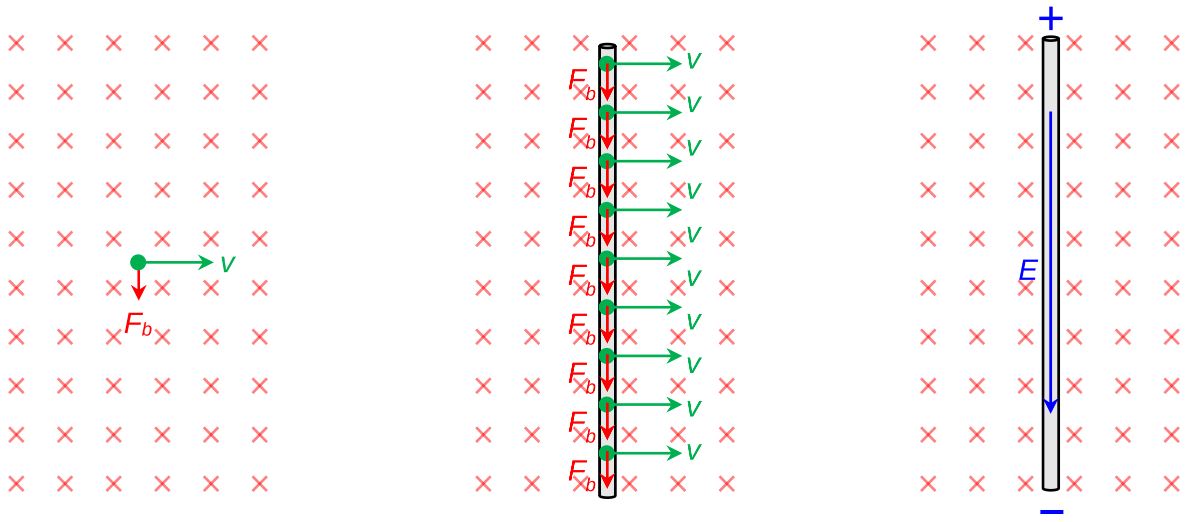
As for the direction of the induced emf, we can use Fleming's Right Hand Rule, where the index finger is *B*, the thumb is *v*, and the middle finger the induced current *I* (in the wire). Do pay attention to the fact that the wire is a "battery", and conventional current flows from the negative terminal to the positive terminal within a battery.

15.1.4.1 Origin of Motional EMF

So you have learnt two types of electromagnetic induction: the "flux changing" type and the "flux cutting" type. But is there any explanation to why an emf is induced in a coil when there is "flux changing" and why an emf is induced in a wire when there is "flux cutting"?

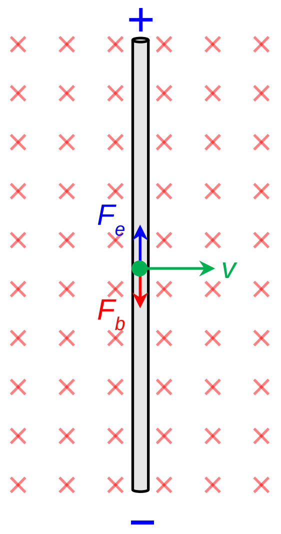
For "flux changing", there simply isn't any explanation. Like we cannot explain why masses attract each other, or why positive charges repel. We don't know why $\varepsilon = -\frac{d\Phi}{dt}$. We just know it happens. "Flux cutting", on the other hand, has a solid explanation. $\varepsilon = BLv$ happens because of $F_b = Bqv$!

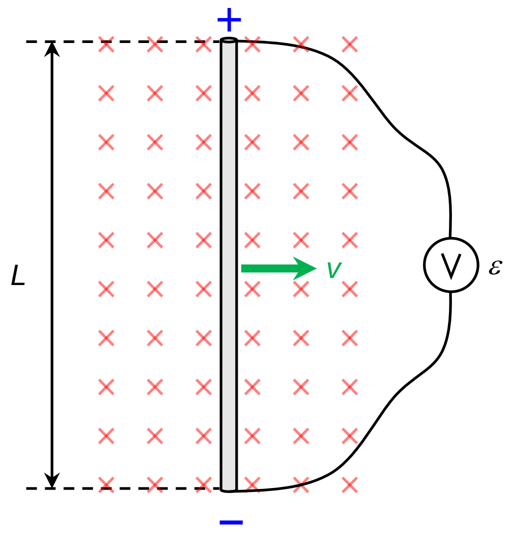
You have learnt that a charge moving in a magnetic field experiences a *Bqv* force. How is this related to a wire moving in a magnetic field? You see, the wire may be neutral in charge as a whole, but it carries a lot of free and mobile electrons! When the wire is moving, the mobile electrons in it (plus the lattice ions) must also move with it. What happens to charges moving in a magnetic field? They experience the $F_b = Bqv$ magnetic force!



electron moving in B *conductor moving in B* *internal E field*

In the diagram above, the wire is moving at speed *v*. So every mobile electron in the wire experiences a downward magnetic force $F_b = Bqv$ that will push them towards the bottom end of the wire. So do ALL the electrons in the rod get pushed all the way down to the bottom end of the wire? No. The reason is because as the electrons accumulate at the bottom end, the bottom end becomes negatively charged, and the top end positively charged. A downward internal electric field *E* develops in the wire.





Now that there is an internal E field, each electron in the wire now experiences an upward electric force of $F_e = qE$ in addition to the downward magnetic force of $F_b = Bqv$. Charge accumulation stops once equilibrium is achieved (instantaneously in practice). And equilibrium is achieved when

$$Bqv = qE$$

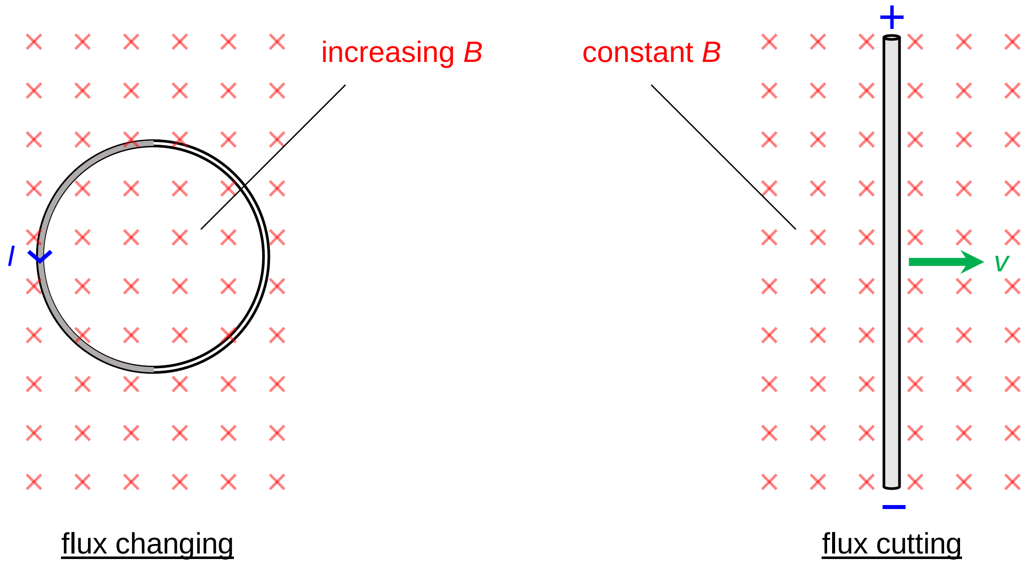
Now since the electric field E is uniform (and $E = -\frac{dV}{dx}$), the potential gradient is simply $\frac{\varepsilon}{L}$, where ε is the potential difference between the top and bottom ends of the wire, and L is the length of the wire. So

$$\begin{aligned} B\cancel{q}v &= \cancel{q}E \\ Bv &= \frac{\varepsilon}{L} \\ \varepsilon &= BLv \end{aligned}$$

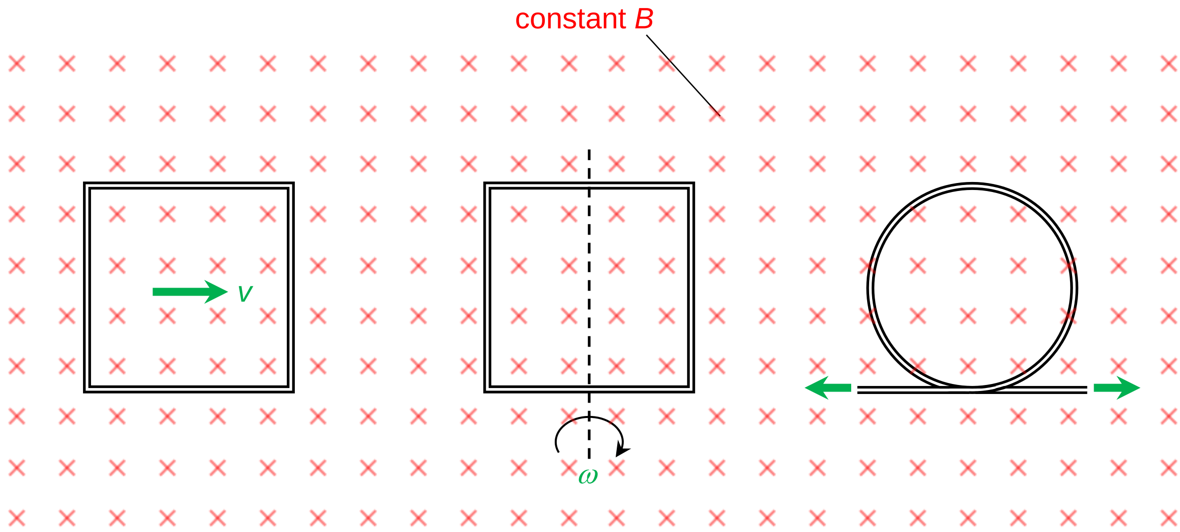
A wire of length L moving across a magnetic field B at speed v behaves like a battery with emf of BLv. Compared to a chemical cell in which charges gain EPE through work done by "chemical" forces, a moving wire is like a "magnetic cell" in which charges gain EPE through work done by magnetic forces. What's cute about this "magnetic cell" is that it must keep moving in order to maintain the emf between its terminals.

15.2 Flux Changing vs Flux Cutting

So there are two parts to Faraday's Laws of Electromagnetic Induction (EMI): the "flux changing" part and the "flux cutting" part. You may ask, so how do we know which one to apply?

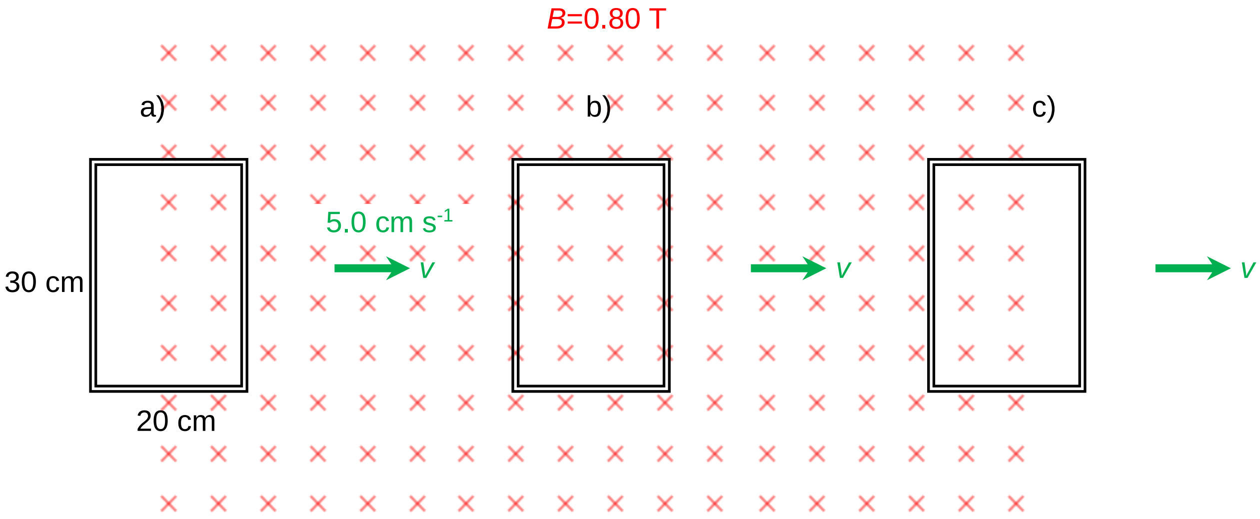


As a rule of thumb, when we see a changing magnetic field (and a stationary coil), we should be thinking of "flux changing" and applying $\varepsilon = -\frac{d\Phi}{dt}$. When we see a moving wire (and a constant magnetic field), we should be thinking "flux cutting" and applying $\varepsilon = BLv$.



How about when we see a coil moving/rotating/deforming in a (constant) magnetic field? As the coil moves/rotates/deforms, its flux linkage can change, which suggests $\varepsilon = -\frac{d\Phi}{dt}$. If we break down the coil into wire segments, the coil is cutting the magnetic flux, which suggests $\varepsilon = BLv$. So should we be using $\varepsilon = -\frac{d\Phi}{dt}$ or $\varepsilon = BLv$? Well, both are ok, as long as you apply them correctly.

Worked Example



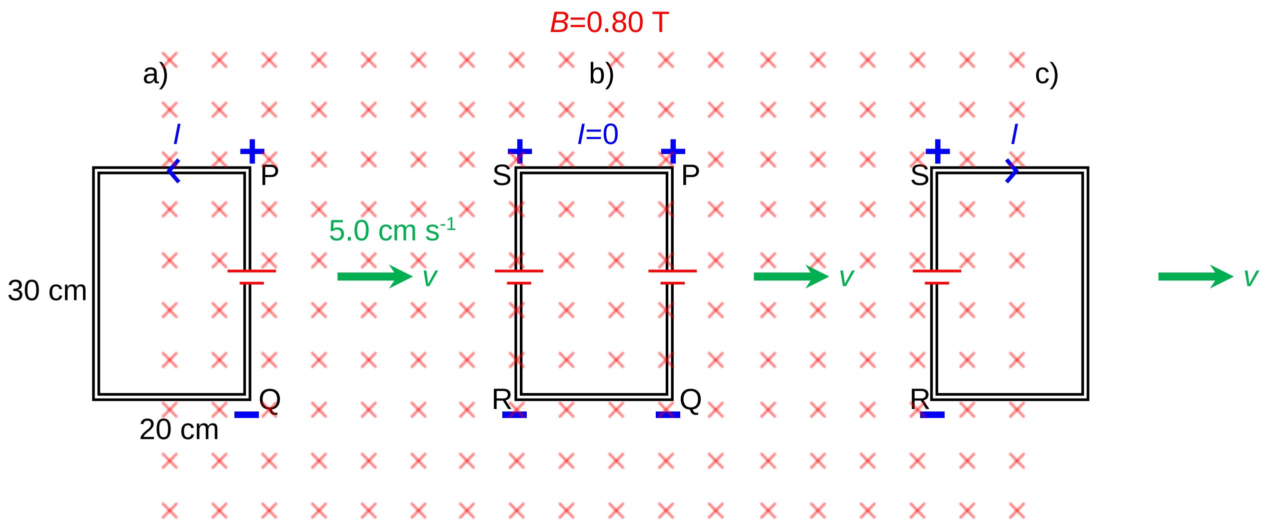
A rectangular coil (30 cm by 20 cm) is moving at a constant speed of 5.0 cm s^{-1}, entering and then leaving a region of uniform magnetic field of 0.80 T. The total resistance of the coil is 6.0 Ω. Determine the direction and magnitude of the induced current in the coil when it is

a) entering,

b) moving within, and

c) leaving the field.

<u>Approach 1: Flux Cutting</u>



a)

As the coil enters the field, only the PQ segment of the coil is cutting the flux (perpendicularly).

Using the FRHR, P and Q are the positive and negative terminals of the "battery" respectively. So the current is going to flow in an anticlockwise direction in the coil.

The emf induced between P and Q is therefore

$$\varepsilon = BLv = (0.80)(0.30)(0.050) = 0.012 \text{ V}$$

The induced current is therefore

$$I = \frac{V}{R} = \frac{0.012}{6.0} = 2.0 \text{ mA}$$

b)

When the entire coil is moving in the field, both the PQ and RS segments of the coil are cutting the flux. But note that PQ is trying to push current in the anticlockwise direction, whereas RS is trying to push current in the clockwise direction. The two emfs "cancel" each other. So the resultant current is zero.

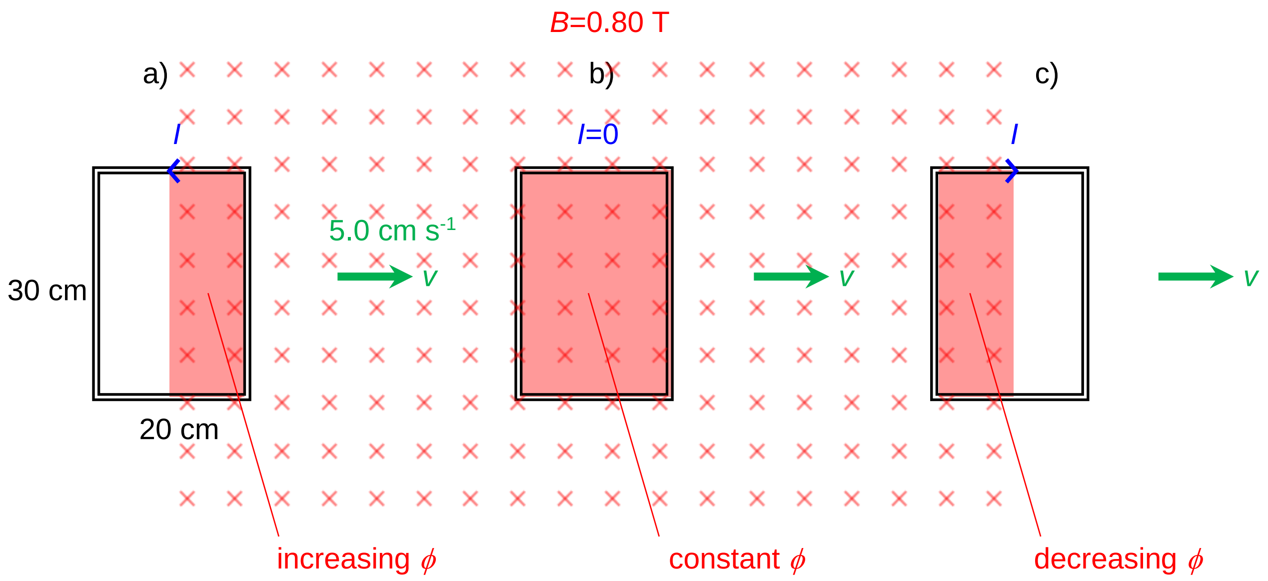
c)

As the coil leaves the field, only the RS segment of the coil is cutting the flux (perpendicularly).

Using the FRHR, R and S are the positive and negative terminals of the "battery" respectively. So the current is going to flow in a clockwise direction in the coil.

The magnitude of the emf and current is the same as part a).

Approach 2: Flux Changing



a)

As the coil enters the field, a larger and larger area is exposed to the magnetic field. So the coil experiences in increasing magnetic flux (going into page). To oppose this change, the induced emf ought to be producing magnetic flux in the opposite direction (coming out of the page) (Lenz's law). Using the RHGR, we can conclude that the induced current is going to be anticlockwise.

Since it takes $20 \div 5.0 = 4.0 \text{ s}$ for the coil to complete the increase in flux,

$$\varepsilon = \frac{d\phi}{dt} = \frac{d(BA)}{dt} = \frac{(0.80)(0.30 \times 0.20)}{4.0} = 0.012 \text{ V}$$

The induced current is therefore

$$I = \frac{V}{R} = \frac{0.012}{6.0} = 2.0 \text{ mA}$$

b)

When the entire coil is moving in the field, the magnetic flux of the coil is constant. Immediately, we can conclude that there is no induced emf and current around the coil.

c)

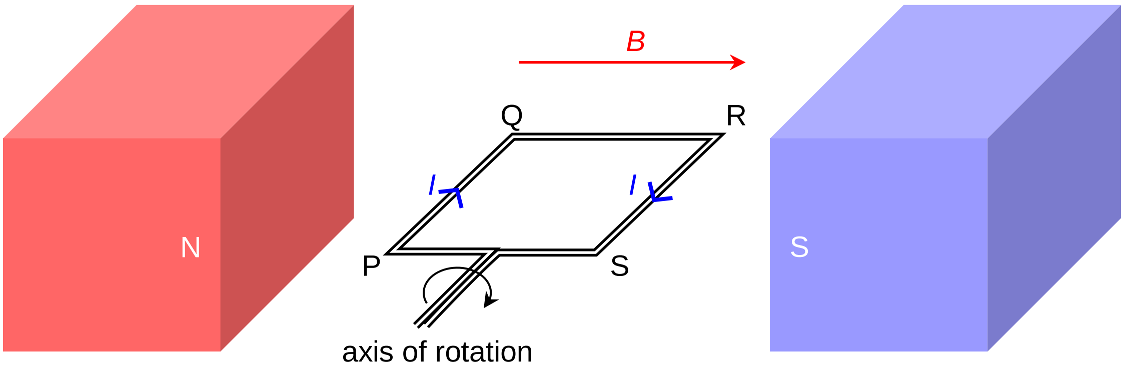
As the coil leaves the field, a smaller and smaller area is exposed to the magnetic field. So it experiences a decreasing magnetic flux (going into the page). To oppose this change, the induced emf ought to be producing magnetic flux in the same direction (going into the page) (Lenz's law). Using the RHGR, we can conclude that the induced current is going to be clockwise.

The magnitude of the emf is same as part a).

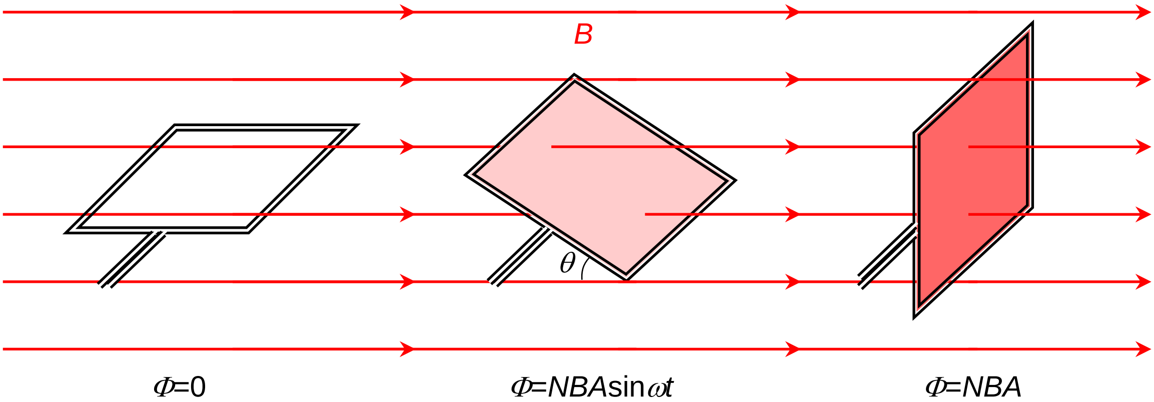
15.3 Generator

The discovery that motion and magnetism when put together can produce electricity led directly to the invention of the electric generator.

Below is the schematic of a very basic electric generator: a rectangular coil (length $L = PQ$ and width $W = QR$) of N turns, rotating in a uniform magnetic field B at constant angular velocity ω.



To determine the direction of the induced emf, it is easier to use the "flux cutting" approach. At this instant, segments PQ and RS are moving vertically upward and downward respectively. Using the FRHR, we realize that both PQ and RS are trying to push current in the clockwise direction.

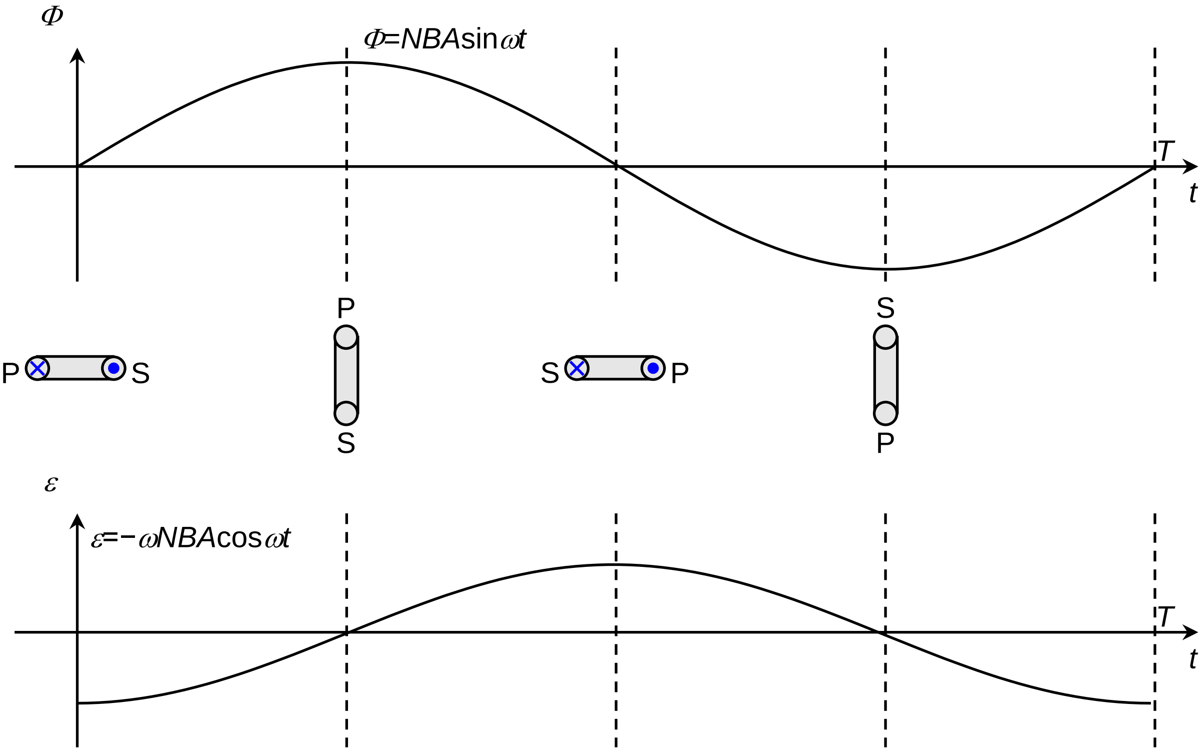


To obtain the magnitude of the induced emf at each orientation, it is actually easier to switch to the "flux changing" approach. First, we note that the flux linkage of the coil is zero when the coil is horizontal, and maximum when it is vertical. In fact, the flux linkage varies sinusoidally.

$$\Phi = NBA\sin\theta = NBA\sin\omega t$$

By considering the rate of change of flux linkage, we obtain

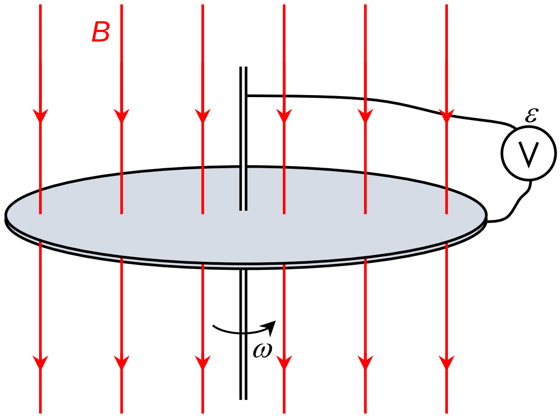
$$\varepsilon = -\frac{d\Phi}{dt} = -\frac{d}{dt}(NBA\sin\omega t) = -NBA\frac{d}{dt}\sin\omega t = -\omega NBA\cos\omega t$$



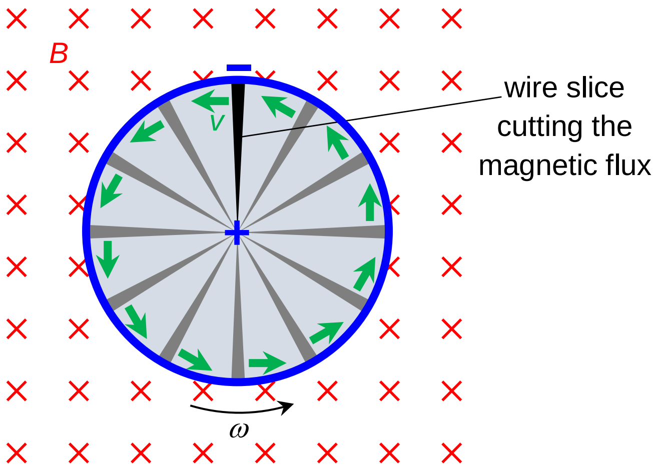
Note that the induced emf also varies sinusoidally (same frequency as the rotation of the coil). Note also that $NBA\omega$ represents the maximum value or amplitude of the induced emf, and it occurs at the instant when the coil is horizontal, when its flux linkage is zero. Note also that the amplitude increases if the coil is rotated at high angular frequency, because of the higher rate of change of flux linkage. (Alternatively, the higher rate of flux cutting).

15.4 Faraday's Disk

The construction of a Faraday's Disk is very simple.

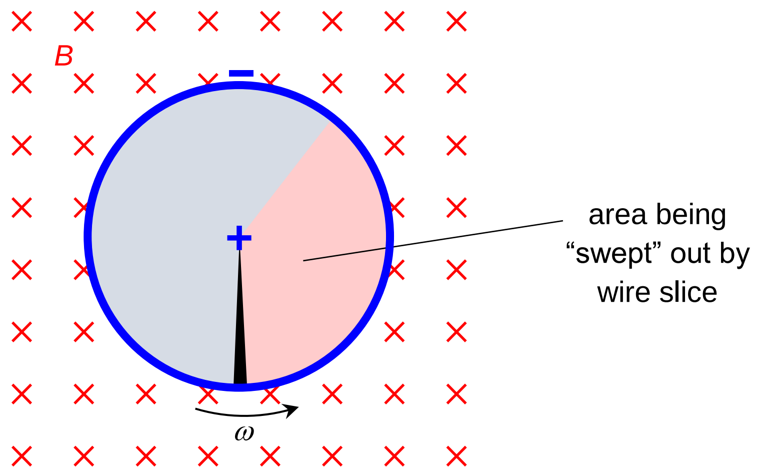


It is nothing more than a disk made of non-ferromagnetic material (e.g. aluminimum) spinning in a (uniform) magnetic field.



Since the field is constant and the conductor is in motion, we should be thinking of "flux cutting". "But I don't see any wire", students complain. Well, we can imagine the disc as consisting of many thin slices of wires, all joined at the centre of the disc, and each extending to the circumference of the disc. As the disc spins, each "slice" will be cutting the magnetic flux.

With this picture in mind, we can then use the FRHR and realize that an emf is induced across each slice such that the centre is the positive terminal, and the circumference is the negative terminal.



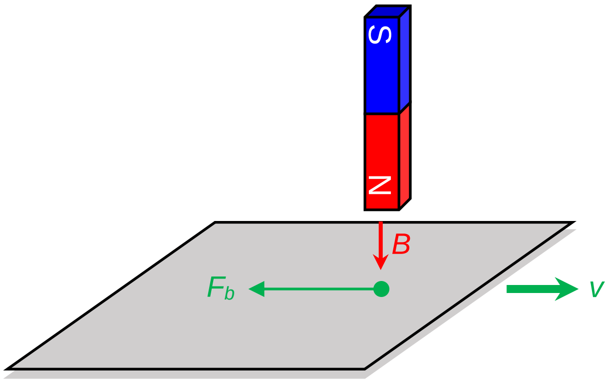
What about the magnitude of the induced emf? We note that each “slice” sweeps out an area equal to the area of the disc $A = \pi r^2$ in one period *T*. So the rate of cutting is

$$\varepsilon = \frac{d\phi}{dt} = \frac{B(\pi r^2)}{T} = BAf$$

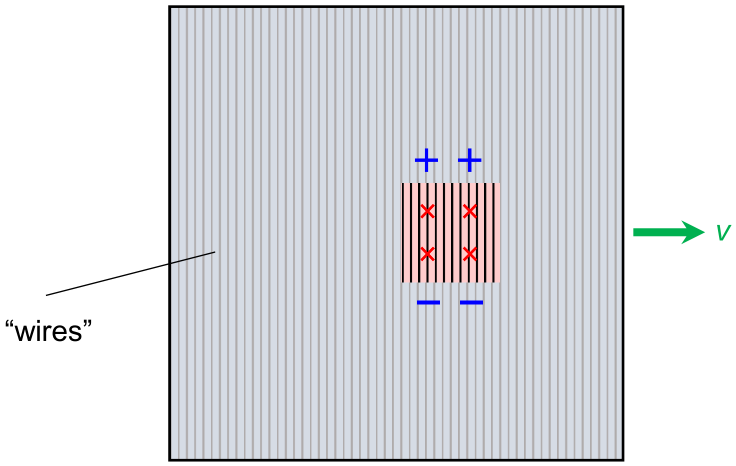
To utilize the emf, we must connect our circuit between the centre and the circumference of the disc. Each slice is an emf source. So the whole disc is basically many emf sources connected in parallel.

15.5 Eddy Current (Magnetic Braking)

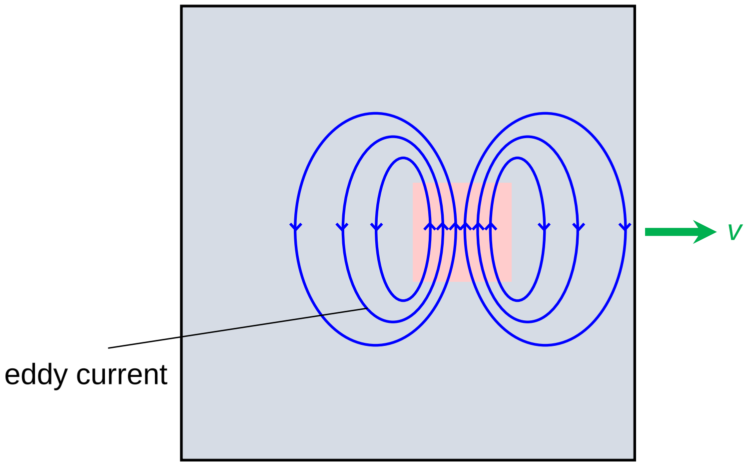
How does a car brake work? A traditional car brake basically clamps the wheel and uses friction to convert the KE of the car into heat energy. Over time, the brake pads and brake discs undergo wear and tear and must be replaced. Wouldn't it be nice if the braking can be achieved without physical contact? Magnetic braking does just that.



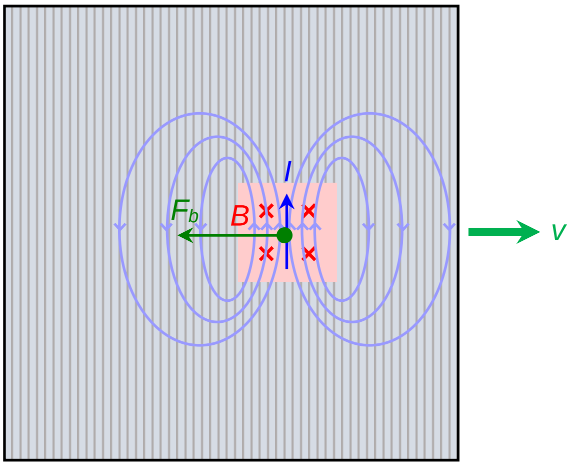
Take for example an aluminum plate sliding under a stationary permanent magnet. Aluminum is a conducting, but non-ferromagnetic material. A permanent magnet will not exert any magnetic force on the plate if it were stationary. It is a different story if the plate were moving. As the plate slides along in the magnetic field, it "cuts" the magnetic flux, resulting in induced emf in the plate. Since aluminum is conducting, an induced current follows. The magnetic field of the induced current in the plate is going to interact with the magnetic field of the permanent magnet. Guess what is going to happen to the motion of the plate? The cause of the induction is the plate's motion. So Lenz's Law dictates that the plate will experience a braking force that slows down the plate to oppose the cause of induction (instead of an accelerating force that reinforces the change).



Now let's look at the scenario in more detail. It is best to think of the plate as being made up of wires. Then as the plate slides along in the magnetic field, only the "wires" passing under the magnet will be "cutting" the magnetic flux. So emf is induced only across the "wires" passing right under the magnet and nowhere else. Those "wires" behave like batteries trying to push current upward (using FRHR).



However, the entire plate is conducting. This results in induced currents that flow in closed loops. We call this kind of current eddy current.



Next, we note that the “wires” under the magnet are now current-carrying conductors moving in a *B*-field. This brings to the table a $F_b = BIL$ magnetic force. Using the FLHR, it is clear that the plate will be experiencing a leftward braking force, an outcome predicted by Lenz’s Law right from the start.

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