XMLECTURE

16 ALTERNATING CURRENT

NO DEFINITIONS. JUST PHYSICS.

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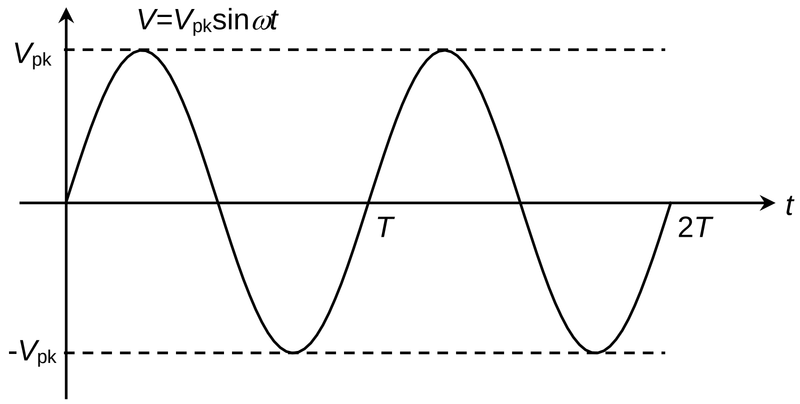
Online resources are provided at https://xmphysics.com/ac

16.1 Alternating Current

During the 1880s, Thomas Edison and Nikola Tesla were embroiled in a battle called the War of the Currents. Edison favoured direct current (DC), with constant voltages and steady currents. Tesla favoured alternating current (AC), with voltages and currents that vary sinusoidally with time. AC emerged victorious in the end, largely because of DC's pathetically low efficiency during power transmission. Throughout the world today, AC power grids deliver power via a sinusoidal emf

$$V = V_{pk} \sin \omega t$$



The voltage and frequency vary from country to country though. Singapore operates on a 230 V (rms) supply voltage and 50 Hz frequency. This implies a peak voltage $V_{pk} \approx 330$ V and angular frequency $\omega = 2\pi f = 2\pi(50) = 100\pi$ rad s^{-1}. So

$$V = 330 \sin 100\pi t$$

16.2 Root-Mean-Square Value

If a fixed resistor R is connected across the AC power supply, the current I flowing in the resistor is also sinusoidal.

$$\begin{aligned} I &= I_{pk} \sin \omega t \\ &= \frac{V_{pk}}{R} \sin \omega t \end{aligned}$$

Since the current through the resistor is not constant, the power dissipated in the resistor also varies with time. Most of the time, we are only interested in the average power $\langle P \rangle$.

Using $\langle \ \rangle$ to denote "the (time) average of", we can write

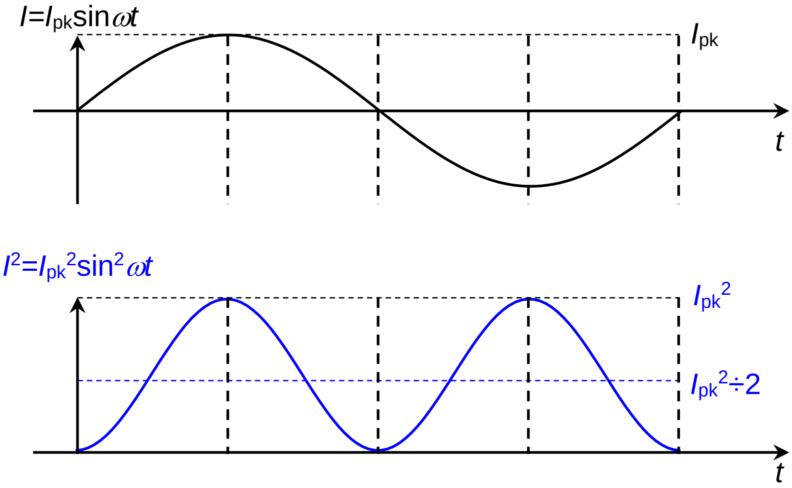
$$\begin{aligned} \langle P \rangle &= \langle I^2 R \rangle \\ &= \langle I^2 \rangle R \\ &= I_{rms}{}^2 R \end{aligned}$$

Note that

- $\langle I^2 \rangle = \frac{1}{T} \int_0^T (I_0 \sin \omega t)^2 \ dt$ is called the mean-square current. It is the average value of I^2 over time.
- $I_{rms} = \sqrt{\langle I^2 \rangle}$ is called the root-mean-square current. It is the square root of the average value of I^2 over time.
- The root-mean-square current is the equivalent constant current that provides the same (average) power. For example, a root-mean-square 3.0 A current passing through a 2.0 Ω resistor results in an average power dissipation of $\langle P \rangle = I_{rms}{}^2 R = (3.0)^2 (2.0) = 18 \text{ W}$. Power wise, this is equivalent to having a constant 3.0 A DC running through the resistor.

For sinusoidal functions, there is a simple relationship between the peak value and the rms value.

$$I_{rms} = \frac{I_{pk}}{\sqrt{2}}$$

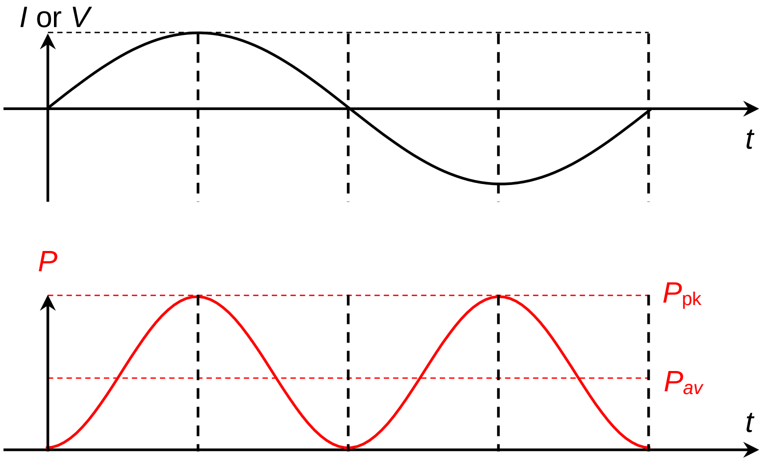


This relationship can be derived simply by inspecting the I^2 graph! As the I^2 value rises and falls between 0 and I_{pk}^2, the symmetry of the sine-square graph makes it obvious that the average value is exactly $\frac{I_{pk}^2}{2}$. If the mean-square value is $\frac{I_{pk}^2}{2}$, the root-mean-square value is of course $\sqrt{\frac{I_{pk}^2}{2}} = \frac{I_{pk}}{\sqrt{2}}$.

We can repeat the exact same argument for a sinusoidal voltage. So for sinusoidal voltages,

$$V_{rms} = \frac{V_{pk}}{\sqrt{2}}$$

16.2.1 Power Dissipation



It's all very simple. With a 50 Hz AC supply, the power dissipation fluctuates at 100 Hz. When it comes to power, the direction of the current does not matter. So there are two power cycles in every one current or voltage cycle.

see video at xmphysics.com

We can also talk about peak power and average power. To calculate peak power, use peak voltage and current values!

$$P_{pk} = I_{pk}{}^2 R = \frac{V_{pk}{}^2}{R} = V_{pk} I_{PK}$$

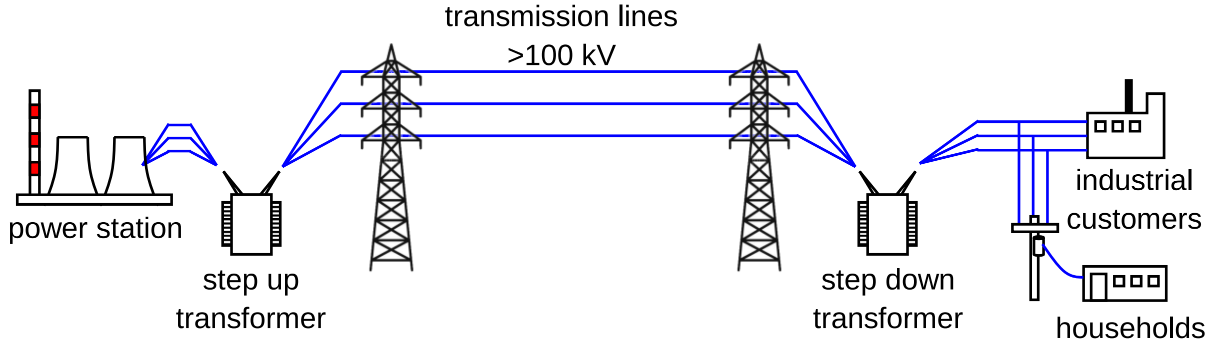
For average power, make sure to use the rms values!

$$\langle P \rangle = I_{rms}{}^2 R = \frac{V_{rms}{}^2}{R} = V_{rms} I_{rms}$$

Or you may want to remember that the peak power is equal to two times the average power (for sinusoidal AC).

$$\langle P \rangle = V_{rms} I_{rms} = \frac{V_{pk}}{\sqrt{2}} \frac{I_{pk}}{\sqrt{2}} = P_{pk} \div 2$$

16.3 Power Transmission



Electric power is generated in power plants, but consumed at households and factories. To connect the two, very long transmission lines (easily hundreds of kilometers of copper cables) are required. Copper may be a fantastic conductor, but over such long distances, the total resistance R is not negligible ($R = \rho \frac{L}{A}$). As a result, the voltage and power lost in transmission cables can be substantial.

If you think in terms of $V = IR$ and $P = I^2R$, it is obvious that the voltage and power lost (in the cable) can be minimized by reducing the current (in the cable). But won't that limit the power supplied (to the end users) as well? Aha, the trick is to use high voltages.

For this reason, the output of electric generators are routinely stepped up to 500 kV on the transmission lines. At the consumer end, the voltage has to be stepped down (rms 230 V for Singapore) for safety and insulation considerations. The device that accomplishes the necessary voltage conversion is called the transformer.

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Worked Example 1

2 MW is to be supplied over a 100 Ω transmission cable. Calculate the power lost and voltage drop in the cable if the transmission line is operated at

a) 500 kV

b) 500 V

Solution

a)

To deliver 2 MW at 500 kV, required current $I = \frac{P}{V} = \frac{2 \times 10^6}{500 \times 10^3} = 4.0 \text{ A}$

Power dissipated in transmission cable $P = I^2 R = 4.0^2 (100) = 1.6 \text{ kW}$

Voltage drop across transmission cable $V = IR = (4.0)(100) = 400 \text{ V}$

If you're alert, you would realize there is a small "error" in our calculation. The power delivered to the customer is less than 2 MW since about 1.6 kW is lost as heat in the cable. Similarly, the voltage that arrives at the customer's doorstep is less than 500 kV, since there is a 400 V drop across the cable. However, because the current is small, the error is negligible.

b)

To deliver 2 MW at 500 V, required current $I = \frac{P}{V} = \frac{2 \times 10^6}{500} = 4000 \text{ A}$

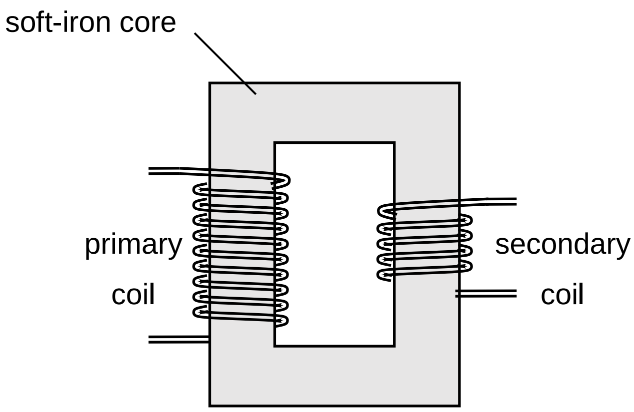
Power dissipated in transmission cable $P = I^2 R = 4000^2 (100) = 1600 \text{ MW}$

Voltage drop across transmission cable $V = IR = (4000)(100) = 400 \text{ kV}$

It is clear that power transmission at low voltages is practically impossible. It basically means that we actually need a voltage of 400.5 kV at the start of the transmission line so that after the 400 kV drop in the cable, the customer gets the 500 V supply that is promised. In delivering 2 MW to the customer, 1600 MW is lost as heat in the cable. Goodness.

16.4 Transformer

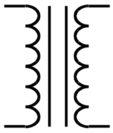
The transformer (despite its exciting sounding name) consists of just two coils of wires wrapped around a soft iron core.



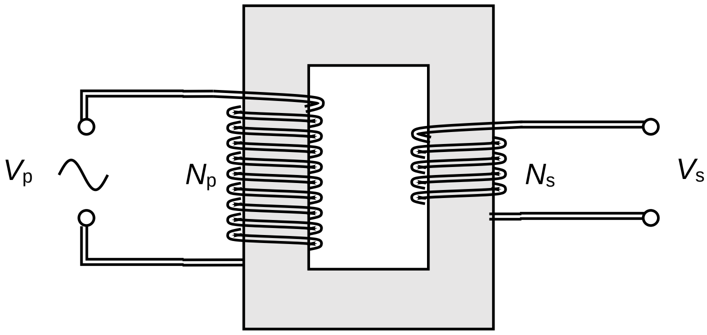
artist impression only.

practical transformer coils have thousands of turns

The coil to which power is supplied is called the primary; the coil from which power is delivered is called the secondary. The circuit symbol for a transformer looks like this



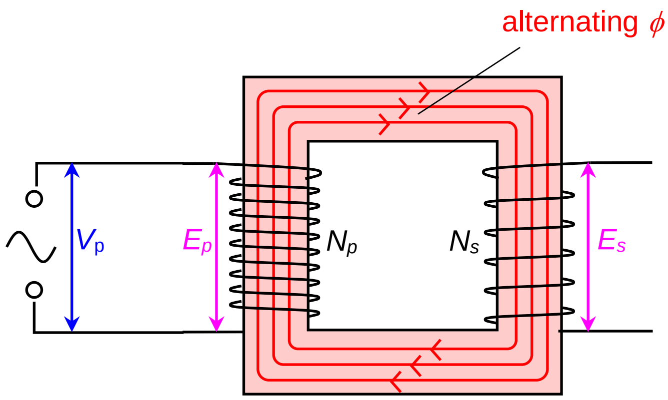
To explain how the transformer works, we will be using V_p and V_s to denote the voltage across the primary and secondary coils respectively. Likewise, N_p and N_s denote the number of turns in the primary and secondary coils respectively.



artist impression only.

practical transformer coils have thousands of turns

Open Circuit Operation



To keep things simple, let's leave the secondary coil open for the time being. When a sinusoidal voltage V_p is applied, the primary coil is magnetized like an electromagnet. This results in an alternating magnetic flux ϕ in the soft iron core.

The primary coil now experiences a changing magnetic flux linkage Φ_p, thanks to the alternating magnetic flux ϕ. Faraday's Law of EMI dictates that an emf E_p is induced in the primary coil[1]:

$$E_p = \frac{d\Phi_p}{dt} = N_p \frac{d\phi}{dt} \quad \cdots\cdots\cdots(1)$$

Since the secondary coil is wrapped around the same core, it is also experiencing the same alternating magnetic flux ϕ. A similar emf E_s is thus induced in the secondary coil:

$$E_s = \frac{d\Phi_s}{dt} = N_s \frac{d\phi}{dt} \quad \cdots\cdots\cdots(2)$$

Dividing the equation (2) by equation (1), we obtain

$$\frac{E_s}{E_p} = \frac{N_s}{N_p}$$

If the primary coil has zero resistance, the self-induced emf E_p must match V_p exactly[2]. Similarly, if the secondary coil has zero resistance, V_s is exactly equal to E_s.

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[1] Did you realize that there is something different about this induction from the ones you have encountered in the EMI chapter? The coil is having an emf induced by its own changing magnetic flux. This is called self-induction, the detail of which is beyond the H2 syllabus.

[2] This should remind you of the back emf in motors. If there is no induced emf, the current through the primary coil would have been infinitely large since it has zero resistance and it is shorted across V_p. But because the induced E_p cancels out the supply voltage V_p (almost completely), the (open circuit) current in the primary coil is a negligibly small current. Often called the magnetizing current I_m, this current is responsible for setting up

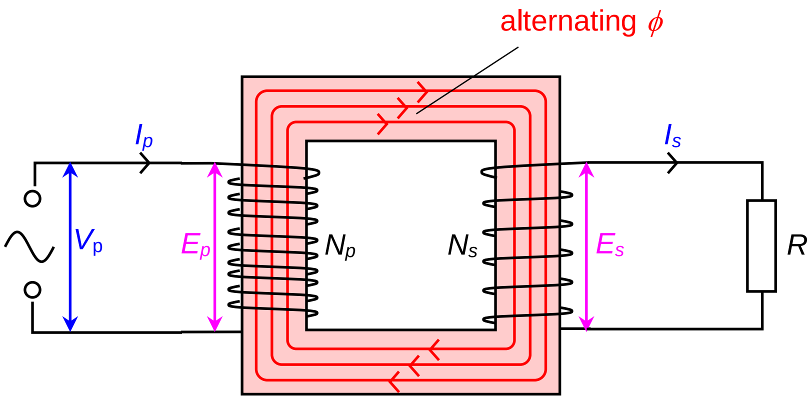
Replacing E_s and E_p with V_s and V_p respectively, we obtain

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

So there we have it: the voltage ratios $\frac{V_s}{V_p}$ follows the turns ratio $\frac{N_s}{N_p}$. A transformer with $\frac{N_s}{N_p} > 1$ is a step-up transformer, since it transforms a lower voltage into a higher one. Conversely, a transformer with $\frac{N_s}{N_p} < 1$ is a step-down transformer.

Closed Circuit Operation

Let's now connect a load R (i.e. a fixed resistor) to the secondary coil.



With the circuit now closed, V_s is going to cause a current I_s to flow in the secondary coil. Ohm's Law applies. So

$$I_s = \frac{V_s}{R}$$

Once I_s is flowing, a current I_p must also flow in the primary coil. Why?

Well, fundamentally it is because of a more exotic form of EMI called mutual induction[3]. But for the H2 level, you're only expected to use the principle of conservation of energy to obtain the relationship between I_p and I_s. Phew.

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the alternating flux in the core, and should be differentiated from the additional current that flows in the primary coil when a load is connected across the secondary coil.

[3] Because the two coils are magnetically connected by the core, an alternating emf and current in one coil would produce an alternating flux which induces an alternating emf and current in the other coil. But isn't the alternating emf and current in this coil also producing an alternating flux which induces the alternating emf and current in the original coil? So who is the inducer and who is the inducee? Both. In fact, both coils are inducing itself and the other and being induced by the other at the same time! Is your head hurting? Relax lah. Detailed understanding of mutual induction is not required by the H2 syllabus.

Firstly, we note that power is delivered to the load R when I_s flows. This power obviously must ultimately come from the power supply connected to the primary coil. In fact, if the transformer is lossless, the power drawn from the supply should be equal to the power delivered to the load. So

$$V_p I_p = V_s I_s$$

From here, we can show that the current ratio is equal to the inverse of the turns ratio.

$$\frac{V_p}{V_s} = \frac{I_s}{I_p} \Rightarrow \frac{I_s}{I_p} = \frac{N_p}{N_s}$$

It is worth highlighting that a step-up transformer steps down the current (by the same factor), and vice versa.

watch video at xmphysics.com

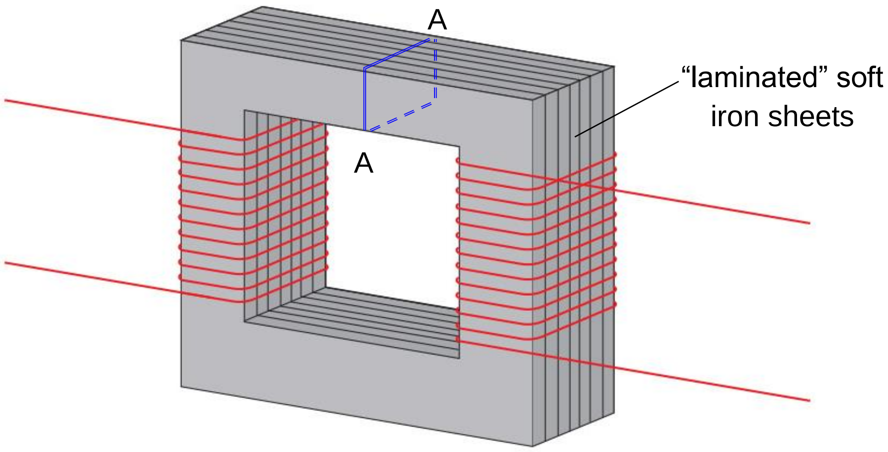
Before we end this section, let's acknowledge the crucial role played by the soft iron core.

- Because soft iron has a very high permeability, the magnetic flux ϕ is increased dramatically. An air-core transformer has no meaningful induction because $\frac{d\phi}{dt}$ is too small.
- Because soft iron has so much higher permeability than air, all the magnetic flux ϕ is completely kept within the core. (Just like electric current flows within a copper wire in an electric circuit, the magnetic flux "flows" within the soft-iron core in this magnetic circuit) This means that the magnetic flux by either coil is completely linked to the other coil, thus maximizing the mutual induction between them.

16.4.1 Eddy Currents

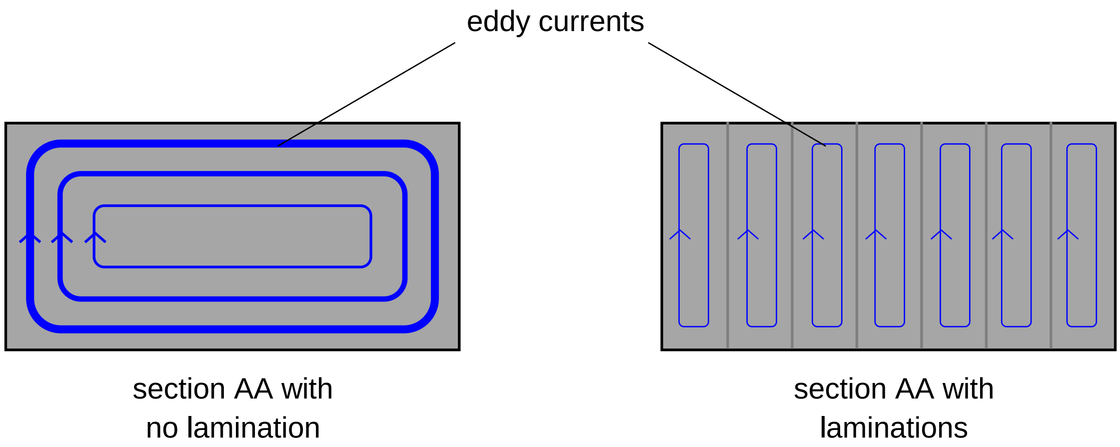
Real transformers always have some energy losses. It is unavoidable for the coils to have some small resistances, resulting in I^2R losses. Energy is also lost through hysteresis in the core[4].

An interesting potential avenue for energy loss in the transformer involves eddy currents. We chose iron for its superior magnetic property, to obtain large (changing) magnetic flux ϕ. But $\frac{d\phi}{dt}$ induces emf, and iron is a good electrical conductor. The resulting eddy currents are very undesirable since they waste energy through I^2R heating.



Fortunately we have a cure for this: laminations. Instead of one solid iron block, we assemble the core from thin iron sheets. These sheets are insulated from one another electrically by either the natural coating of oxide or insulating varnish. While the permeability to the flow of magnetic flux in the core is preserved, the conductivity to the flow of electric current (in the perpendicular plane) is severely reduced.

[4] Energy is required to change the orientation of the magnetic domains in the soft iron. Hysteresis is not in the H2 syllabus.



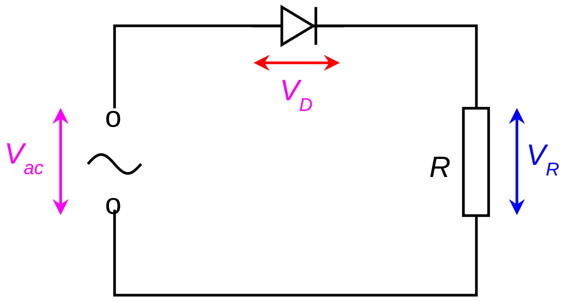
This is because the eddy currents are now constrained to flow within each thin sheet. The resistance is greatly increased (because of reduced cross sectional area and $R = \rho\frac{L}{A}$), and the magnitude of the eddy current greatly reduced!

16.4 Rectification

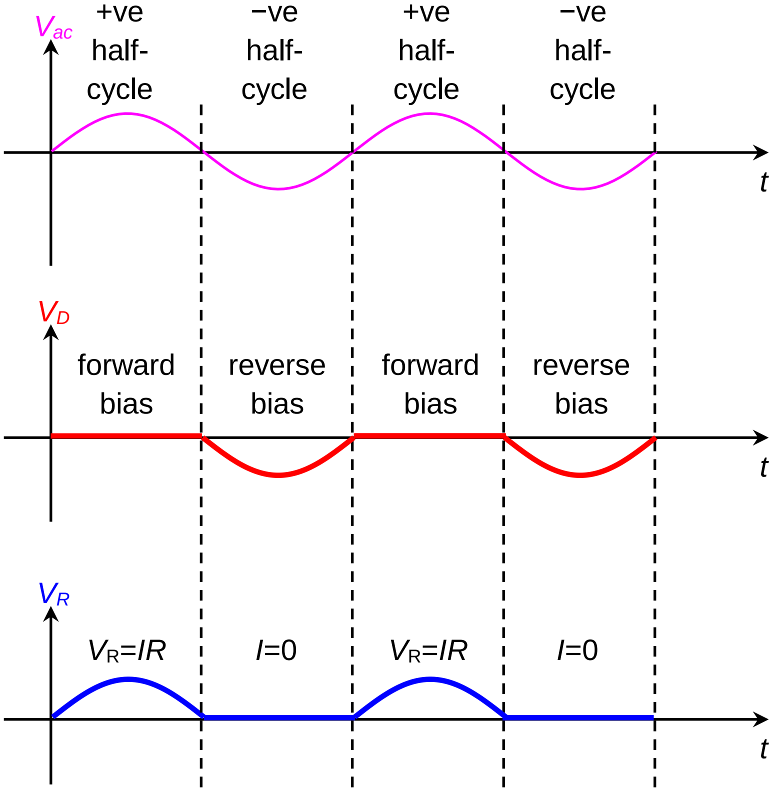
The computer chips are designed to operate with a steady and constant supply voltage. That's why they must be powered by DC. What if you only have an AC power supply? Aha. That's what power adapters are for.

16.4.1 Half Wave Rectification

If all we want is to remove the voltage and currents in the "wrong" direction, we only need one single diode.



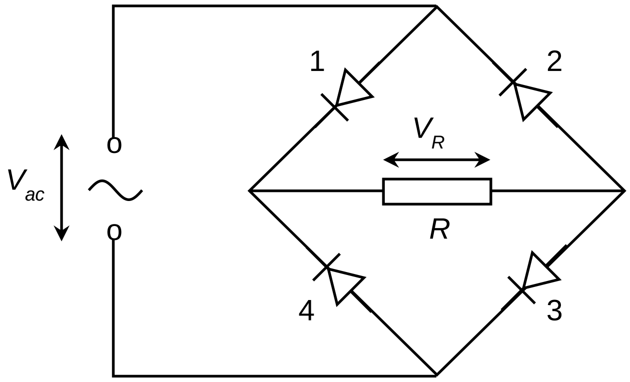
Remember that a diode can be turned on or off, depending on whether it is in forward or reverse bias? So a diode is perfect for the job of connecting the resistor *R* to the AC supply during the +ve half cycles, and disconnecting it during the −ve half cycles.



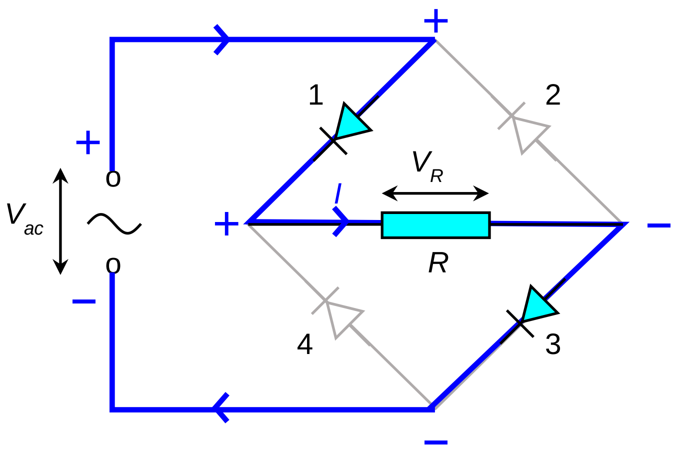
This is called half wave rectification. It is as if the AC supply is switched on only half the time. Obviously, the power supplied is halved. Compared to the full AC whose rms value is $\frac{V_{pk}}{\sqrt{2}}$, the rms value for the half-wave rectified AC is $\frac{V_{pk}}{\sqrt{2}} \div \sqrt{2} = \frac{V_{pk}}{2}$. But is there any way to avoid sacrificing half the available power? The answer is in the next section.

16.4.2 Full Wave Rectification

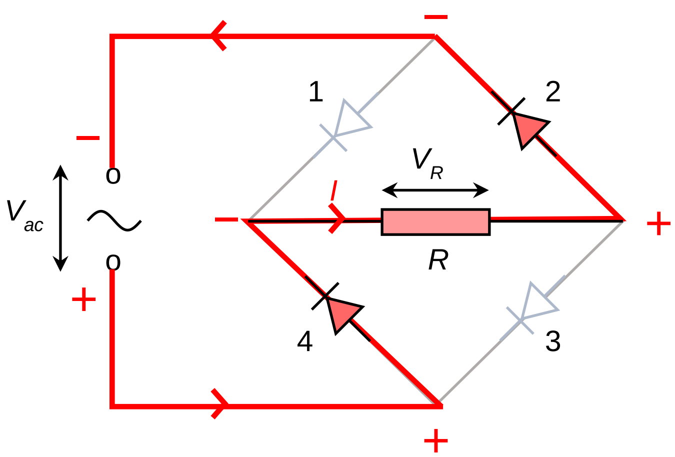
Full wave rectification requires four diodes.



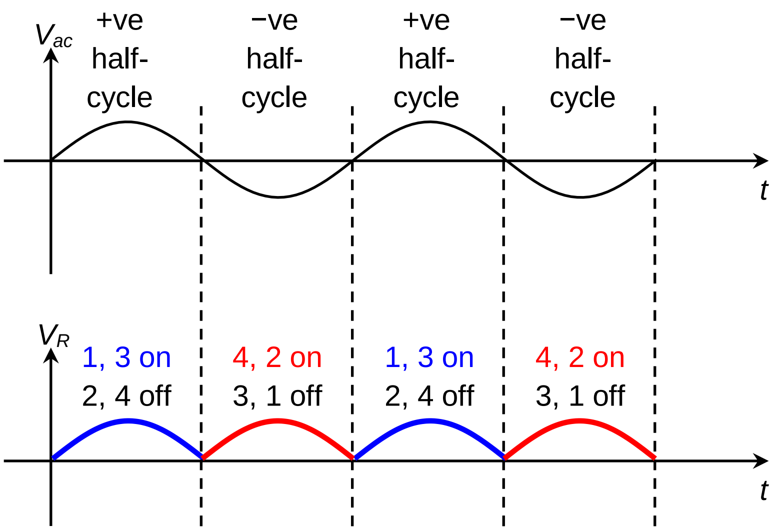
Working in pairs, the diodes take turns to switch on and off, allowing current to pass through the resistor all the time. As such, current is drawn from the AC supply during both positive and negative half cycles, yet the current passing through the resistor does not switch direction!



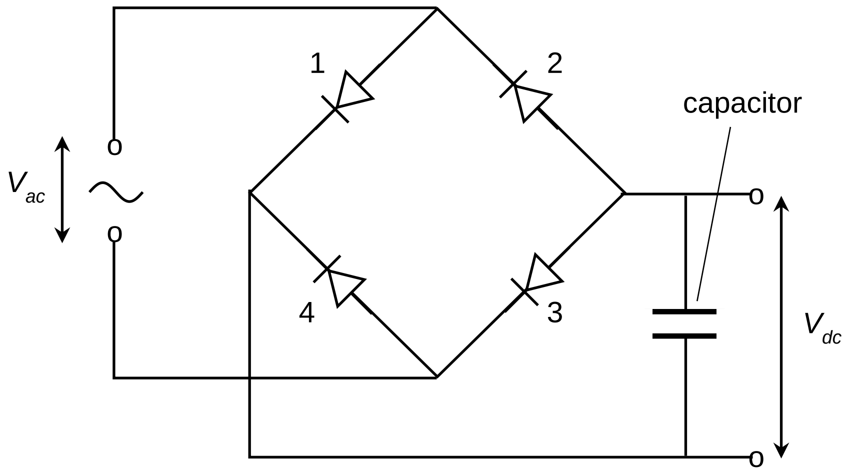
During +ve half-cycle

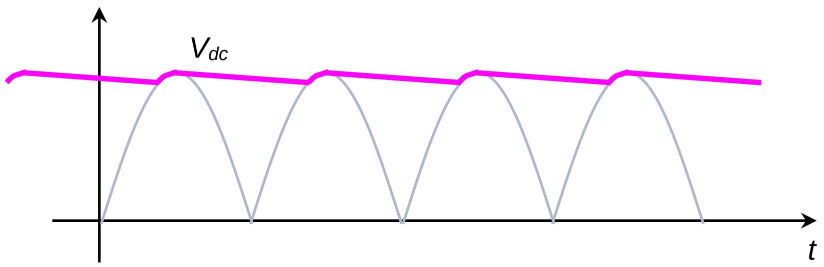


During −ve half-cycle



So we have managed to "flip" (instead of remove) the negative half cycles. Yay. But we still cannot power computer chips with this humpy voltage because the chips will be powered off every time the voltage drops to zero. What's missing is a capacitor[5] to smooth out the humps to achieve a constant DC voltage (with a little bit of ripple). But I'll just mention it in passing since capacitors are not in the H2 syllabus.





[5] A capacitor is an electrical component that is able to store charges. Capacitance is not in the H2 syllabus.