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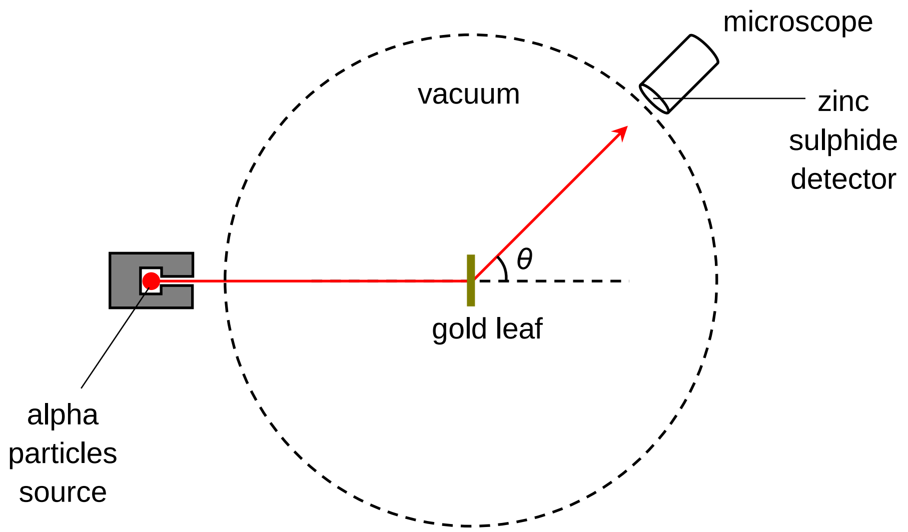
18.1.0 Properties of Nuclei

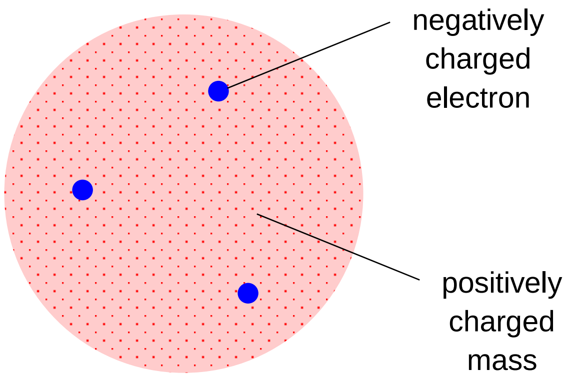
In 1911, Rutherford instructed his assistant Geiger, who instructed the graduate student Marsden to perform the experiment that has come to be known as the Rutherford scattering experiment. Why didn't Rutherford and Geiger conduct the experiment themselves? Probably because they expected it to be a tedious and exhausting experiment with no interesting results. To Marsden's credit, he conducted the experiment meticulously and resiliently. And the results blew everyone's mind.

18.1.1 Rutherford Alpha-Particle Scattering Experiment

The basic setup of this experiment involves firing a stream of alpha particles at a very thin gold foil. A microscope with a glass screen coated with zinc sulphide is used to detect the landings of alpha particles at different deflection angles θ. (Marsden had to spend hours squinting into the microscope, for each value of θ, counting the flashes each time an α-particle impacts the screen at that angle)



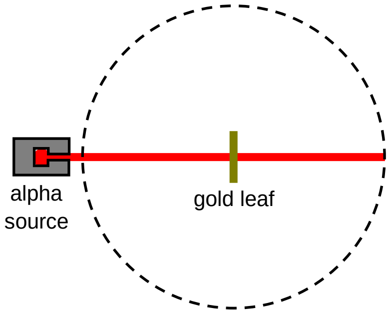
At the time of experiment, the prevailing model for the atom was the Plum Pudding Model, proposed by J J Thomson after he discovered that the electron is a component of every atom. It was believed that other than the electrons, the rest of the atom (i.e. the mass and the positive charge) was distributed uniformly throughout the volume of the atom.



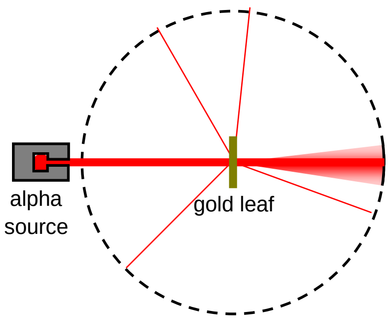
Plum Pudding Model of the Atom

Based on calculations of the known volume, mass and charge of the atom, nobody expected the alpha particles (which were known to be massive and moving at very high speeds) to experience any meaningful deflection by the few gold atoms they encounter as they cruise through the thin gold leaf[1].

However, actual experimental observations showed that even though most (> 99%) of alpha particles passed through with little or no angular deflection, a few alpha particles (about 1 in 8000) suffered deflections of more than 90°. A very small number were even deflected backwards!



Expected Results



Actual Observations

Without a doubt, those particles had encountered an electrostatic force far greater than Thomson's model suggested they would. These results were so unexpected that Rutherford later wrote: "It was quite the most incredible event that ever happened to me in my life. It was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it came back and hit you."

Rutherford realized that a much more concentrated positive charge is required to produce such a strong electric repulsion on the alpha particles. To understand this, recall that the strongest electric field strength of a (uniformly) charged sphere is at the surface of the sphere: the smaller the sphere, the stronger the field. For example, the maximum field strength of a 1 uC of charge distributed evenly

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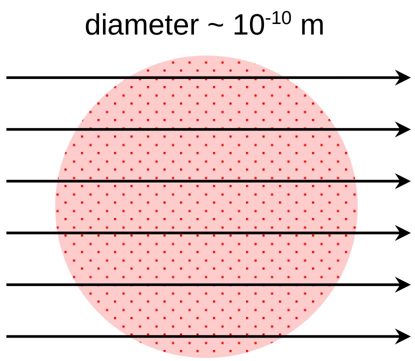
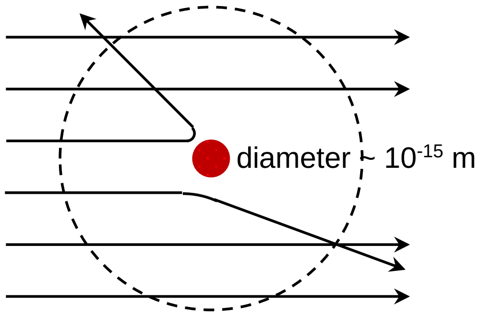
[1] Gold was the chosen material because it could be rolled out to be very very thin without breaking.

in a spherical volume of radius 1 m is $\frac{1}{4\pi\varepsilon_0}\frac{1\times10^{-6}}{1^2}$ N C^{-1}. The same amount of charge if squeezed into a sphere of radius 1 mm would be $\frac{1}{4\pi\varepsilon_0}\frac{1\times10^{-6}}{0.001^2}$ N C^{-1}, one million times stronger!



Plum Pudding Model



Rutherford Model

Rutherford worked out the required size of the nucleus to produce the distribution of deflection angles observed in the experiment. The conclusion was astounding: instead of being distributed over a sphere of radius 10^{-10} m (the estimated size of an atom), the positive charge (and the remaining mass of the atom) had to be concentrated in a tiny sphere of radius 10^{-15} m! This very dense particle (both in mass and charge) came to be known as the **nucleus**.

This is a shocking revelation. Firstly, the nucleus occupies only a tiny fraction $(\frac{10^{-15}}{10^{-10}})^3 = 10^{-15}$ of the volume of the atom. Since everything is made of atoms, everything is 99.9999999999999% empty by volume! Secondly, the density of the nucleus (2.3×10^{17} kg m^{-3}) is out-of-this-worldly high. At such density, a raindrop 1 mm in diameter weighs about 120,428 tons (heavier than an aircraft carrier). Truth is indeed stranger than fiction.

18.1.2 Neutrons Protons Electrons

In 1917, Rutherford discovered that firing alpha particles at nitrogen gas atoms results in the production of hydrogen nuclei. He concluded that these positively charged hydrogen nuclei must have originated from the nuclei of the nitrogen atoms. This marked the discovery of the **proton**.

Meanwhile, there was speculation that there was something else in the nucleus. For example, the helium nucleus was known to have the charge of 2 protons but the mass of roughly 4 protons. Rutherford suggested that the additional mass must be due to an uncharged particle in the nucleus. This particle was eventually named the **neutron** when Chadwick proved its existence in 1932.

Finally, the components of an atom have been confirmed: electrons, protons and neutrons. Since then, we have made very precise measurements of their mass and charge.

	mass	charge
neutron	$m_n = 1.008665u$ $= 1.67 \times 10^{-27}$ kg	0
proton	$m_p = 1.007276u$ $= 1.67 \times 10^{-27}$ kg	$+e = 1.60 \times 10^{-19}$ C
electron	$m_e = 0.000548580u$ $= 9.11 \times 10^{-31}$ kg	$-e = 1.60 \times 10^{-19}$ C

Note that

- The **unified atomic mass unit** u is defined to be the mass of $\frac{1}{12}$ the mass of a neutral carbon-12 atom. Since one mole of carbon-12 atoms has a mass of exactly 12 g,

$$1u = \frac{1}{12}(0.012 \div 6.02 \times 10^{23}) = 1.66 \times 10^{-27} \text{ kg}$$

- The electron has only 1/1833 times the mass of a proton, but carries the exact same magnitude of charge.
- The neutron is approximately the same mass as the proton, but carries zero charge.
  - More precisely, the neutron is a tiny wee bit more massive than the proton. It kind of makes sense later when you learn that a neutron can actually "split" to become a proton plus an electron.

18.1.3 Nuclides

Since the neutrons and protons make up the nucleus, they are also called **nucleons**.

In chemistry, an element refers to atoms with the same number of protons. In nuclear physics, a **nuclide** refers to atoms with the same number of protons and neutrons. In other words, all atoms of a particular nuclide have the same nucleus composition.

A nuclide is identified using the A-Z notation:

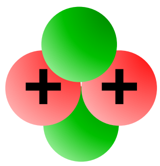
$$ {}^{A}_{Z}X $$

A, the **mass number** (aka the nucleon number) states the number of nucleons in the atom.
Z ,the **atomic number**, states the number of protons in the nucleus. It is also known as the proton number or (even more meaningfully) the charge number.
X, the chemical symbol, states the element of the atom.

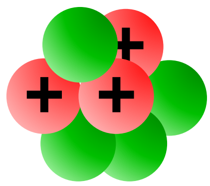
For example, ${}^{14}_{6}C$ refers to carbon atoms with 14 nucleons and 6 protons (and thus $14-6=8$ neutrons). Since the name of the element (carbon) determines the atomic number, *Z* is sometimes omitted, as in ${}^{14}C$, or simply carbon-14.

Illustrated below are the nuclei of the ${}^{1}_{1}H$, ${}^{4}_{2}He$ and ${}^{7}_{3}Li$ nuclides.

hydrogen-1



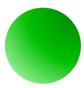
helium-4



lithium-7

The A-Z notation is also used to denote the proton ${}^{1}_{1}p$, the neutron ${}^{1}_{0}n$ and the electron ${}^{0}_{-1}e$.

proton



neutron

electron

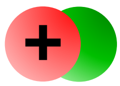
Isotopes refer to atoms with the same atomic number *Z* but different mass number *A*, i.e. atoms with the same number of protons but different number of neutrons. For example, carbon has 15 known isotopes. The four main ones are tabulated below.

	abundance	atomic mass	half-life	decay mode	decay product
$^{11}_{6}C$	trace	11.011433*u*	20 min	β+	boron-11
$^{12}_{6}C$	98.9%	12*u*	stable	-	-
$^{13}_{6}C$	1.1%	13.003355*u*	stable	-	-
$^{14}_{6}C$	trace	14.003241*u*	5730 y	β-	nitrogen-14

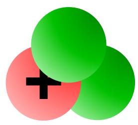
Note that isotope have the same chemical properties, but different physical properties. For example, both $^{12}_{6}C$ and $^{14}_{6}C$ undergo the exact same chemical combustion with oxygen to form carbon dioxide. However, $^{12}_{6}C$ is a stable nucleus whereas $^{14}_{6}C$ radioactively decays to form nitrogen-14. $^{12}_{6}C$ and $^{14}_{6}C$ also differ in mass and can be separated using mass spectroscopy.

Illustrated below are the nuclei of three hydrogen isotopes: $^{1}_{1}H$, $^{2}_{1}H$ and $^{3}_{1}H$.

proton



deuteron



triton

18.2.0 Nuclear Binding

Chemistry students are familiar with the concept of bond energy which kind of measures how strong a chemical bond is. In nuclear physics, we talk about binding energy, which kind of measures how strongly the nucleons in the nucleus are bound together.

18.2.1 Mass-Energy Equivalence

From the special theory of relativity, Einstein realized that apart from the kinetic energy due to its motion, a particle also has energy of mc^2 even when it is resting (see Appendix A if you're interested). This energy came to be known as the rest energy.

From here, Einstein argues that the mass of a system m (when it is at rest) is a manifestation of the total energy of the system. This includes the KE of any moving constituents (e.g. atoms) in the system, the energies of any photons trapped in the system, and any form of potential energy whether positive and negative. In short, if a system has (rest) mass m, then the system has total (rest) energy

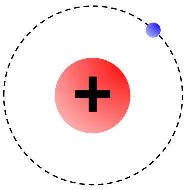
$$E = mc^2$$

This idea came to be known as the **mass-energy equivalence** principle.

Let's think of some implications of this principle:

1. When a torchlight is switched on, it is emitting photons and thus losing energy continuously. Shouldn't its mass be decreasing continuously then?
2. Hot water has higher internal energy than cold water (mainly due to the higher KE of the vibrating H2O molecules). Shouldn't hot water weigh more than cold water then?
3. Shouldn't the mass of an ice cube be lower than the mass of the water that formed it since the H2O molecules in the ice cube has lower PE?
4. To ionize a hydrogen atom, energy must be supplied. Does this not mean that the mass of an individual proton and electron is higher than the mass of a hydrogen atom?



Based on Einstein's equation, the change in mass can be calculated by $\Delta m = \frac{\Delta E}{c^2}$. So let's crunch the numbers for our four examples.

	System	Energy gained/lost	Change in mass $\Delta m = \frac{\Delta E}{c^2}$
1	Shining a 1 W torchlight for 1 min	$1 \times 60 = 60$ J	$\frac{60}{(3.00 \times 10^8)^2} = 6.67 \times 10^{-16}$ kg
2	Heating up 1 kg of water by 1 °C	4200 J	$\frac{4200}{(3.00 \times 10^8)^2} = 4.67 \times 10^{-14}$ kg
3	Freezing 1 kg of water	33600 J	$\frac{33600}{(3.00 \times 10^8)^2} = 3.73 \times 10^{-13}$ kg
4	Ionizing 1 mole of hydrogen atoms	$N_A \times 13.6$ eV	$= \frac{(6.02 \times 10^{23})(13.6 \times 1.60 \times 10^{-19})}{(3.00 \times 10^8)^2} = 1.46 \times 10^{-11}$ kg

The (percentage) change in mass, as predicted by theory, is clearly too small to be measured for ordinary energy transactions. Our everyday experiences are not able to tell us whether the mass of an object is related to the energy of the object. So where can we possibly find supporting evidence for the mass-energy equivalence principle? The answer lies in the atomic nucleus.

18.2.2 Mass Defect and Binding Energy

Let's use the helium-4 nucleus as our example.

$${}^{4}_{2}He$$

Since He-4 consists of 2 protons and 2 neutrons, its mass is roughly 4u. But look carefully at the data:

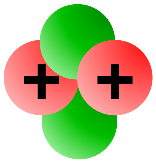
Mass of 2 protons and 2 neutrons $= 2m_p + 2m_n$

$= 2(1.008665u + 1.007276u)$

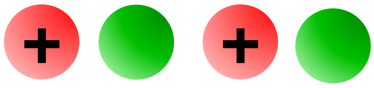
$= 4.03188u$

Mass of 1 helium-4 nucleus $= 4.00151u$

Did you notice that the mass of a He-4 is smaller than the mass of the 2 protons and 2 neutrons that formed it?



4.00151u



4.03188u

The difference between the mass of a nucleus and the mass of its constituent nucleons is called the **mass defect**, Δm. In other words, for a ${}^{A}_{Z}X$ nuclide,

$$\Delta m = (Zm_p + Nm_n) - m_X$$

For the He-4,

$$\Delta m = (2m_p + 2m_n) - m_{He}$$
$$= 2(1.008665u + 1.007276u) - 4.00151u$$
$$= 0.03037u$$

Consider the system consisting of the 2 protons and neutrons. When the protons and neutrons were at infinite distances apart from one another, the system is at a higher energy level compared to when the protons and neutrons are bound in a helium nucleus[2]. From the mass-energy equivalence principle, the mass defect is merely a reflection of the loss of energy by the system.

[2] Well, the nuclear potential energy of the system has changed from zero to some negative value. This is similar to the situation of an electron becoming bound to a proton in a hydrogen atom, or the Moon becoming trapped in the Earth's gravitational field; instead of negative EPE due to the attractive electrical forces and negative GPE due to the gravitational attraction, we are dealing with negative nuclear potential energy due to the attractive nuclear forces.

It's time to introduce this quantity called the binding energy (*BE*): the energy required to separate a nucleus completely into its constituent neutrons and protons. The *BE* of a nucleus is related to its mass defect by the formula

$$BE = \Delta m.c^2$$

For the helium-4 nucleus,

$$\begin{aligned} BE &= (0.03037)(1.66\times10^{-27}).(3.00\times10^{8})^2 \\ &= 4.53\times10^{-12} \text{ J} \\ &= 28.3 \text{ MeV} \end{aligned}$$

So why is it called the binding energy? You see, when a helium-4 nucleus is formed by fusing 2 protons and 2 neutrons, 28.3 MeV of energy is released, as reflected by the mass defect of 0.03037*u* in the He-4 nucleus. Logically, to reverse the process completely, 28.3 MeV of energy must be supplied to the helium-4 nucleus to separate it back into the 2 protons and 2 neutrons. When we say that He-4 has a binding energy of 28.3 MeV, we actually mean that 28.3 MeV is required to unbind it. For many physics students (especially those who do not take chemistry), binding energy takes a bit of time to get used to. Because *BE* of a nucleus is not energy that the nucleus possesses. It is the energy that has already been released, and which is now "owed" by the nucleus. *BE* is actually a kind of energy deficit, or negative potential energy.

Anyway, the *BE* of a nucleus depends on its nuclear composition, so each nuclide will have its own *BE*. The larger the mass defect of a nucleus, the larger its binding energy.

Nuclide	*A*	*Z*	*N*	(1) Mass of nucleons/*u*	(2) Mass of nucleus/*u*	(1)−(2) Mass Defect/*u*	*BE*/MeV
Deuterium-2	2	1	1	2.015941	2.013553	0.002388	2.23
Helium-4	4	2	2	4.031882	4.001585	0.030297	28.29
Lithium-7	7	3	4	7.056488	7.013489	0.042999	40.15
Beryllium-9	9	4	5	9.072429	9.010175	0.062254	58.13
Iron-56	56	26	30	56.449126	55.921961	0.527165	492.24
Lead-206	206	82	124	207.671092	205.933721	1.737371	1622.27
Polonium-210	210	84	126	211.702974	209.941089	1.761885	1645.16
Uranium-235	235	92	143	236.908487	234.998126	1.910361	1783.80
Uranium-238	238	92	146	239.934482	238.005026	1.929456	1801.63

Tabulated above is the mass defect and binding energy of a few nuclides[3].

Note that:

- *BE* is usually in the MeV range, which is many orders of magnitude larger than the energies we deal with in electronic transitions (eV) or even X-ray (keV).
- By definition, a free proton or a free neutron has zero *BE*, since they are already the constituents.

In conclusion, the mass defect of nuclides provide evidence for the mass-energy equivalence principle. The associated binding energies are of practical importance because they allow us to calculate the energies of nuclear reactions, as you shall see in the next section.

[3] Actually, what's published is usually the mass of a neutral atom (electrons included), instead of the mass of the nucleus (stripped of all electrons). So I had to work backwards to figure out the mass of the bare nucleus to compile this table.

18.3.0 Nuclear Reactions

While chemical reactions involve the rearrangement of atoms (to form new compounds), nuclear reactions involve the rearrangement of nucleons (to form new nuclei). While chemical reactions are represented symbolically by chemical equations, nuclear reactions are represented by nuclear equations.

Take a look at this one.

$$ {}^{4}_{2}He + {}^{14}_{7}N \rightarrow {}^{17}_{8}O + {}^{1}_{1}H $$

Historically, this was the nuclear reaction that led to the discovery of the proton. Rutherford was firing alpha-particles (which are very energetic He-4 nuclei) at nitrogen atoms (N-14). Amazingly, oxygen (O-17) and hydrogen nuclei (which are just protons) were then detected.

Let's do some calculations using the following given data:

Mass of ${}^{4}_{2}He = 4.002\ 602u$ Mass of ${}^{17}_{8}O = 16.999\ 133\ u$

Mass of ${}^{14}_{7}N = 14.003\ 074u$ Mass of ${}^{1}_{1}H = 1.007\ 825\ u$

Mass of reactant nuclei before the reaction $\sum m_i = 4.002602u + 14.003074u = 18.005676u$

Mass of product nuclei after the reaction $\sum m_f = 16.999133u + 1.007825u = 18.006958u$

Apparently, there is an increase in mass after the reaction.

$$ \begin{aligned} \Delta m &= \sum m_f - \sum m_i \\ &= 18.006958u - 18.005676u \\ &= 0.001282u \end{aligned} $$

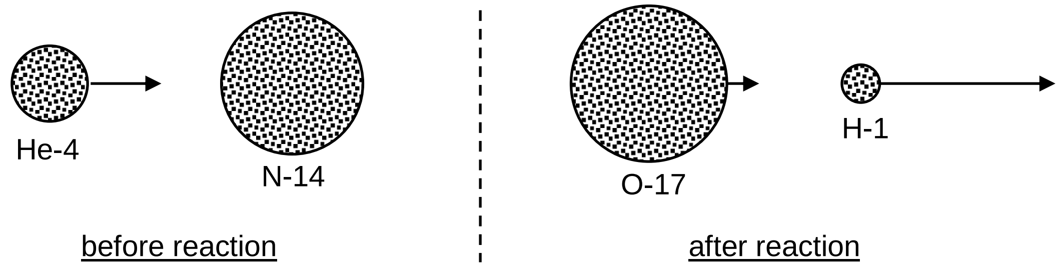
By the mass-energy equivalence principle, the increase in mass implies that energy is absorbed by the system during the reaction. And the amount of energy is

$$ \begin{aligned} \Delta E &= \Delta m.c^2 \\ &= (0.001282)(1.66 \times 10^{-27}).(3.00 \times 10^{8})^2 \\ &= 1.915 \times 10^{-13} \text{ J} \\ &= 1.20 \text{ MeV} \end{aligned} $$

[4]

[4] Note that Δm here does not denote the mass defect (of any specific nuclide). It merely denotes the change in mass during the nuclear reaction.

Who is supplying this required energy for this reaction to happen? The alpha particles. This reaction actually requires the alpha particles to have at least 1.20 MeV of KE. (In comparison, the nitrogen atoms have practically zero KE). This 1.20 MeV is required to re-arrange the nucleons from the lower energy configuration of the reactant nuclei into the higher energy configuration of the product nuclei. This explains why this reaction does not occur spontaneously in our atmosphere. The hydrogen atoms in the atmosphere are not energetic enough to force this nuclear reaction (with the nitrogen atoms they encounter).



If you're very vigilant, you will realize that 1.20 MeV is actually not enough. Why? Notice that the momentum of the system before the reaction is non-zero (due to the momentum of the alpha particles). By PCOM, the momentum should remain non-zero after the reaction. This means that the oxygen and hydrogen nuclei must be "born" with some KE, so to speak. The alpha particles must therefore provide for the KE of the reactants in addition to the 1.20 MeV required to re-arrange the nucleons. In other words, the alpha-particle must have KE of at least 1.20 MeV + *X*, where *X* represents the KE of the reactants. If not this reaction cannot go ahead. Get it?

18.3.1 Nuclear Fusion

Nuclear fusion occurs when two smaller atomic nuclei combine to form a larger nucleus.

For example, a deuteron (H-2) can combine with a triton (H-3) to create a helium nucleus (He-4), releasing one neutron.

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n$$

Let calculate the energy released during this fusion.

BE of $^{2}_{1}H = 2.224573$ MeV BE of $^{4}_{2}He = 28.295673$ MeV

BE of $^{3}_{1}H = 8.481821$ MeV BE of $^{1}_{0}n = 0$

Total BE before fusion $\sum BE_i = 2.224573 + 8.481821 = 10.706394$ MeV

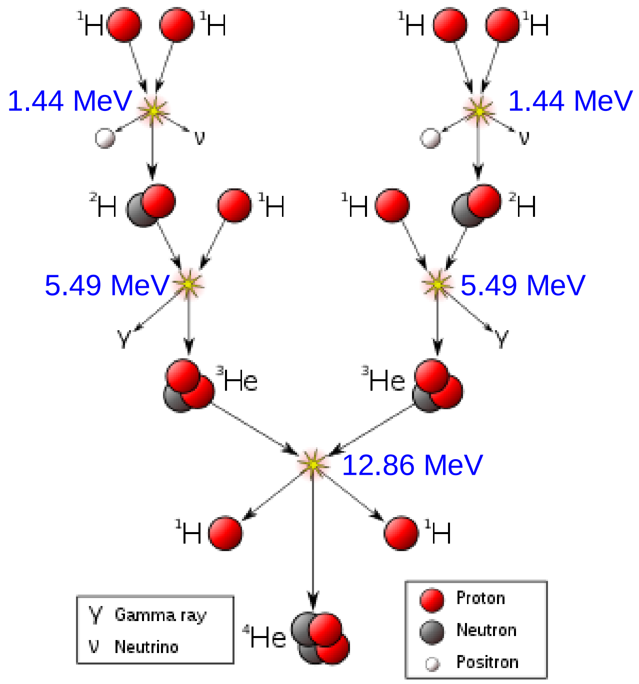
Total BE after fusion $\sum BE_f = 28.295673 + 0 = 28.295673$ MeV

Increase in BE, $\Delta BE = \sum BE_f - \sum BE_i$

$= 28.295673 - 10.706394$

$= 17.6$ MeV

An increase in total BE implies lower total rest energy. The complete logic chain is that an increase in total BE implies an increase in total mass defect (since $BE = \Delta m.c^2$), which implies a decrease in total mass. So an increase in total BE of 17.6 MeV implies that 17.6 MeV of energy is released during a deuterium-tritium fusion. This energy is mainly carried by the KE of the He-4 and the neutron.



Our Sun (and all stars) generates its energy from the hydrogen fusion series: the larger nuclei formed from each stage serve as the inputs to the next stage. So hydrogen combine to form deuterium to form tritium to form helium, and tremendous amounts of energy is released at each stage. You may wonder why hydrogen fusion does not occur among the hydrogen atoms in our atmosphere. Well, in order for fusion to occur, the nuclei have to be brought within "touching distance" of each other. Unfortunately, the electrical repulsion between the two protons in the hydrogen nuclei is formidable.

Assuming the radius of a proton to be 10^{-15} m, the electrical potential energy for two protons to be at "touching distance" is about

$$\frac{1}{4\pi\varepsilon_0}\frac{(1.60\times10^{-19})(1.60\times10^{-19})}{1\times10^{-15}} = 1.44 \text{ MeV}$$

1.44 MeV represents a lot of KE for a hydrogen nucleus. Even the hydrogen nuclei in the Sun's core, at temperature of 15×10^{6} K, need a little help from quantum tunneling to achieve fusion. On Earth, we have yet to develop a physically and economically viable process to generate power using fusion. Which is a pity, because nuclear fusion promises an even cleaner and safer source of energy compared to nuclear fission.

18.3.2 Nuclear Fission

Nuclear fission occurs when a large atomic nucleus splits into two smaller nuclei of approximately the same size. Of particular interest to us is the fission of uranium-235, since this is the reaction that powers our nuclear plants (and also the atomic bomb dropped on Hiroshima).

When U-235 nuclei are bombarded by neutrons, one of the many fission reactions it can undergo is the following:

$$ {}_0^1n + {}_{92}^{235}U \rightarrow {}_{56}^{141}Ba + {}_{36}^{92}Kr + 3\,{}_0^1n $$

Let's calculate the energy released in this reaction.

BE per nucleon of ${}_{92}^{235}U = 7.591$ MeV

BE per nucleon of ${}_{56}^{141}Ba = 8.326$ MeV

BE per nucleon of ${}_{36}^{92}Kr = 8.513$ MeV

Total BE before fission $\quad \sum BE_i = 235 \times 7.591 = 1783.9$ MeV

Total BE after fission $\quad \sum BE_f = 141 \times 8.326 + 92 \times 8.513$

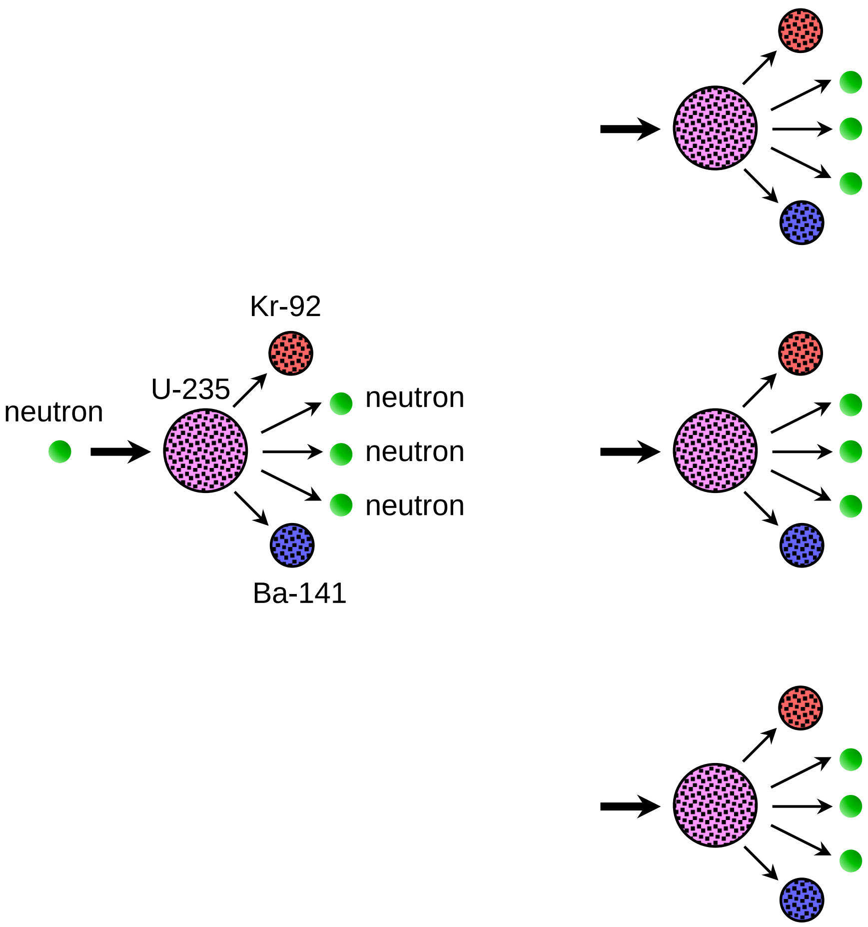
$$= 1957.2 \text{ MeV}$$

Increase in BE, $\Delta BE = \sum BE_f - \sum BE_i$

$$= 1957.2 - 1783.9$$

$$= 173 \text{ MeV}$$

The increase in BE of 173 MeV implies that 173 MeV of energy is released during the fission. This energy is carried by the KE of the fission products.



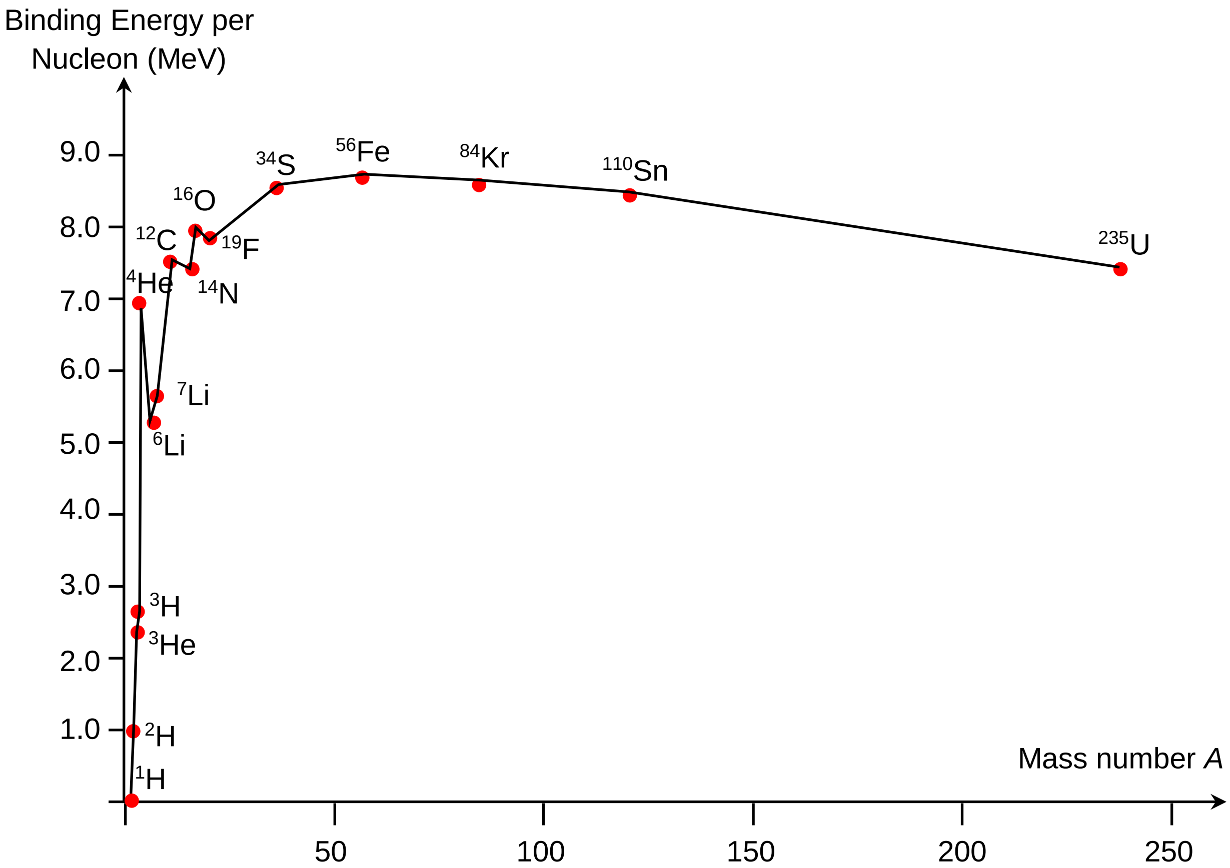
Conveniently, the fission products include 3 neutrons. If these 3 neutrons strike 3 other U-235, they can trigger 3 more fissions. This results in a chain reaction that can be self-sustaining in the nuclear reactor. The power output of nuclear reactors is adjusted using control rods containing elements (boron, cadmium, etc) which absorb neutrons strongly. If uncontrolled, the exponential nature of the chain reaction results in a nuclear explosion, which is of course disastrous. Having said that, in the only two nuclear disasters in the past (meaning the Chernobyl and the Fukushima), the meltdown was not caused by the nuclear fission of the fissile fuel, but the radioactivity of the fission products (such as Ba-141 and Kr-92 in this example), which forms the subject of section 18.4.

18.3.3 Binding Energy per Nucleon

Besides the binding energy *BE*, another important parameter for a nuclide is the binding energy per nucleon *BE*/*A*. As the name suggests, *BE*/*A* is simply the *BE* of a nuclide divided by the number of nucleons *A* in that nuclide. *BE*/*A* turns out to be a good indicator of the stability of a nuclide, which kind of makes sense since *BE*/*A* roughly represents the energy to remove one nucleon from the nucleus.

Nuclide	*A*	*BE* (MeV)	*BE*/A (MeV)
Deuterium-2	2	2.23	1.12
Helium-4	4	28.29	7.07
Lithium-7	7	40.15	5.74
Beryllium-9	9	58.13	6.46
Iron-56	56	492.24	8.79
Lead-206	206	1622.27	7.88
Polonium-210	210	1645.16	7.83
Uranium-235	235	1783.80	7.59
Uranium-238	238	1801.63	7.57

An interesting trend presents itself when *BE*/*A* is plotted against *A*.



Note that:

- Iron-56, with the highest *BE*/*A* of 8.8 MeV, is recognized as the most strongly bound and most stable nuclide in the world.
- The general trend is that *BE*/*A* increases with *A* for small nuclei (*A*<56) but decreases with *A* for large nuclei (*A*>56).[5]
- When small nuclei undergo fusion, the product nuclei tend to have higher *BE*/*A* than the reactant nuclei. A good example is the deuterium-tritium fusion.

$$ {}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n $$

- Conversely, when large nuclei undergo fission, the product nuclei tend to have higher *BE*/*A* than the reactant nuclei. A good example is the uranium fission.

$$ {}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{0}n $$

- Since the number of nucleons remain the same, a higher *BE*/*A* also implies higher total *BE*. This explains why energy is released by the fusion of small nuclei (e.g. hydrogen), and the fission of large nuclei (e.g. uranium).

---

[5] If you want to know the reason behind this trend, and why He-4 and O-16 are outliers, see Appendix B:Nuclear Force and C:Magic Numbers.

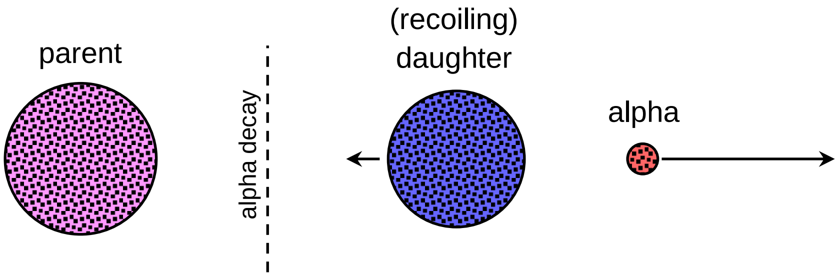
18.4.0 Radioactivity

Radioactive decay occurs when an unstable nucleus spontaneously (and randomly) transforms into a different nucleus, emitting different kinds of radiation in the process. The H2 syllabus requires knowledge of just three kinds of decays: alpha, beta and gamma.

18.4.1 Alpha Decay

An alpha particle is actually a very energetic helium-4 nucleus. So an alpha decay is represented by the equation

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$



The parent nucleus *X*, transforms itself into the (smaller) daughter nucleus *Y*, after ejecting the helium-4 nucleus. The alpha particle speeds off at very high speed in one direction while the daughter nucleus recoils in the opposite direction.

<u>Why are alpha particles mono-energetic?</u>

Let's use the alpha decay of Radon-222 into Polonium-218 as an example.

$$^{222}_{86}Rn \rightarrow {}^{218}_{84}Po + {}^{4}_{2}He$$

Atomic mass of Rn-222 $= 222.0176u$

Atomic mass of Po-218 $= 218.0090u$

Atomic mass of He-4 $= 4.0026u$

Firstly, let's consider the energy of the reaction. Obviously, the total kinetic energy of the decay products come from decrease in rest energy. From the mass-equivalence principle, this can be calculated from the decrease in mass.

$$\begin{aligned}\text{Total } KE_{total} &= \Delta m.c^2 \\ &= [222.0176 - (218.0090 + 4.0026)](1.66\times10^{-27}).(3.00\times10^{8})^2 \\ &= 8.964\times10^{-13}\text{ J} \\ &= 5.603\text{ MeV}\end{aligned}$$

Next, we bring in momentum considerations. Since the total momentum before the reaction is zero (Rn-222 was at rest), it should remain zero after the reaction. This means the momentum of He-4 p_α in one direction must be balanced by the momentum of Po-218 p_{po} in the opposite direction. (This is the exact scenario as the rifle and bullet).

$$p_\alpha = p_{po} = p$$

If we use the $KE = \frac{p^2}{2m}$ formula, we can very quickly work out the ratio of the KE between the daughter nucleus and the alpha particle.

$$\frac{KE_{po}}{KE_\alpha} = \frac{\cancel{p}_{po}^{\ 2}}{2(218u)} \div \frac{\cancel{p}_\alpha^{\ 2}}{2(4u)}$$
$$\frac{KE_{po}}{KE_\alpha} = \frac{4}{218}$$
$$KE_\alpha = 54.5 KE_{po}$$

Using the "model mathematics" you learnt in primary school if you must, we can see that

$$KE_\alpha = \frac{54.5}{54.5+1} KE_{total} = 0.982 KE_{total}$$

$$KE_{po} = \frac{1}{54.5+1} KE_{total} = 0.018 KE_{total}$$

If you run through our working, you will realize that all the alpha particles produced by the decay of radon-222 must have the same KE. Why this is so?

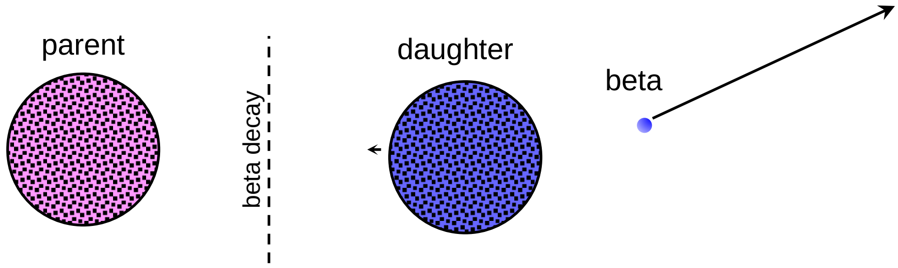
a) The total energy to be shared by Po-218 and He-4 is fixed (because PCOE constrains them to be equal to the energy released by the decay).
b) The ratio of their kinetic energy is fixed by their mass ratio (because PCOM constrains them to have equal but opposite momentum).
c) Therefore, the fraction of the total energy each particle gets is fixed. In fact, Po-218 is many times larger than He-4, it gets only a paltry 1.8% of the total energy. At 98.2%, the bulk of the energy goes to the KE of the alpha particle.

Even though we have used radon-222 as an example, the results are applicable for all alpha decays. Alpha particles (produced by a particular nuclide) are mono-energetic. And the bulk of the energy is with the alpha particles.

18.4.2 Beta Decay

A beta particle is basically a very energetic electron. A beta decay is represented by the equation

$$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e$$



If you compare the *A* and the *Z* numbers of the parent and daughter nuclei carefully, you will realize that the nucleus has lost one neutron, gained one proton and thrown out one electron. What the heck happened? Well, a neutron has decided to split itself into a proton and an electron. The electron is ejected at high speed (as the beta particle), while the proton stays behind in the nucleus.

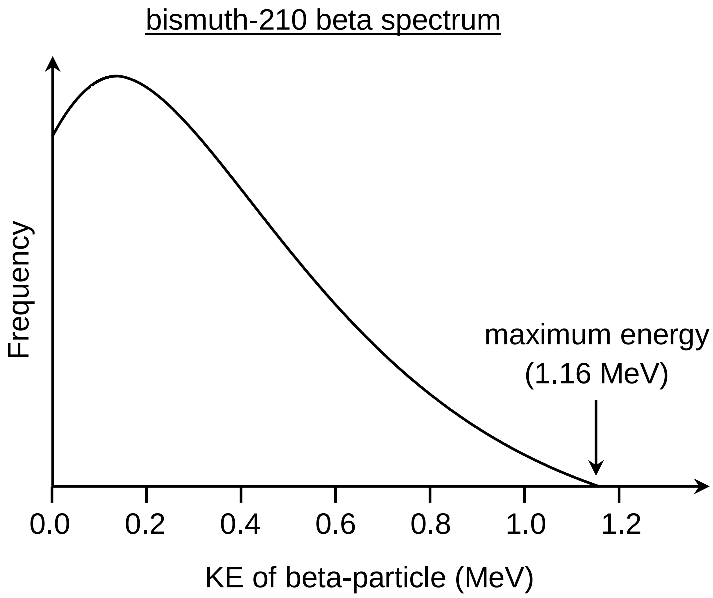
<u>The Neutrino</u>

Let's use the beta-decay of bismuth-210 as an example. After one of its neutrons has changed into a proton, bismuth changed its chemical identity to polonium.

$$^{210}_{83}Bi \rightarrow {}^{210}_{84}Po + {}^{0}_{-1}e$$

The total mass of a polonium-210 and an electron is smaller than the mass of a bismuth-210, as is expected of any nuclear reaction that releases energy. From the mass difference, the energy released can be calculated to be a respectable 1.16 MeV.

Since Po-210 is almost 400,000 times as massive as the electron, from momentum consideration, we would expect the daughter nucleus to show negligible recoil and carry negligible KE. This means that all the beta particles should have KE of 1.16 MeV.



However, it was observed that the beta particles produced by bismuth-210 had a continuous energy spectrum (unlike alpha radiation, which are mono-energetic). Furthermore, the recoil of the polonium-210 nucleus is not always in opposite direction to the beta particle. OMG, what happened to the principles of conservation of energy and momentum? Physicists were so stuck that Niels Bohr actually proposed it was time to dump the most fundamental conservation laws.

In 1931, Wolfgang Pauli suggested humorously that there was probably an as yet unobserved particle emitted during a beta decay. He gave this particle a very cute sounding name, neutrino, which means the little neutral one. Having no charge and almost negligible mass, the neutrino is practically undetectable. However, having this "imagined" particle to carry some of the "missing" energy and momentum does provide a solution to the beta decay puzzle and save PCOM and PCOE.

So the beta process was updated to include the emission of a neutrino.

Beta-minus decay: $\,^{A}_{Z}X \rightarrow \,^{A}_{Z+1}Y + \,^{0}_{-1}e + \bar{\upsilon}$

The symbol $\bar{\upsilon}$ denotes an anti-neutrino. They had to do this because they have given the neutrino to the positive beta decay.

Beta-positive decay: $\,^{A}_{Z}X \rightarrow \,^{A}_{Z-1}Y + \,^{0}_{1}e + \upsilon$

The symbol υ denotes a neutrino, and $\,^{0}_{1}e$ is a positron.

The beta-decay in the H2 syllabus refers to the negative beta decay only. You are also not required to know anything about the neutrino, other than the context in which its existence was predicted.

By the way, the elusive neutrino evaded experimental detection until 1956, a remarkable 26 years after its "discovery".

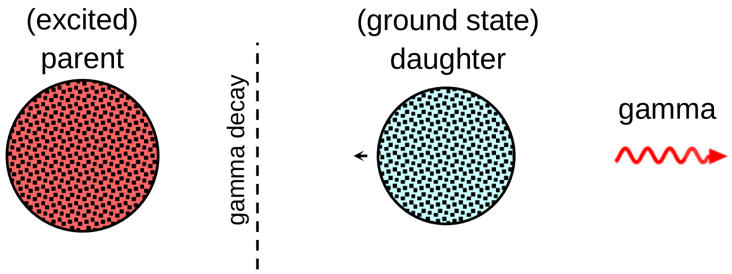
18.4.3 Gamma Decay

Gamma radiation is actually gamma photons.

Sometimes, an alpha or beta decay produces a daughter nucleus in an excited state.

It turns out that just like atoms, nuclei also have quantized energy levels. Because the nuclear forces in the nucleus are so strong, the gaps between the nucleus' energy levels are measured in MeV. So when an excited nucleus de-excites, the transition from the excited energy level to the ground level procures a gamma photon.

$${}_{Z}^{A}X^{*} \rightarrow {}_{Z}^{A}X + \gamma$$



So during a gamma decay, there is no change in the nuclear composition. It simply changes from an excited nucleus (denoted by *) to a ground state nucleus, releasing the energy in a gamma photon (denoted by γ)

18.4.4 Properties of Radiation

When radioactivity was first discovered around 1896, physicists were able to identify three distinct types of radiation due to their different properties. At that time, nuclei, electrons and photons were still unknown. So these three radiations were given the names α, β, γ (the first three letters of the greek alphabets).[6]

	alpha particles	beta particles	gamma ray
Ionizing Power	very strong	strong	weak
Penetrating Power	low	medium	high
Stopped by	paper	~mm aluminum	~cm lead
Range in Air	~cm	~m	∞
Deflection in E-field	yes	yes	no
Deflection in B-field	yes	yes	no

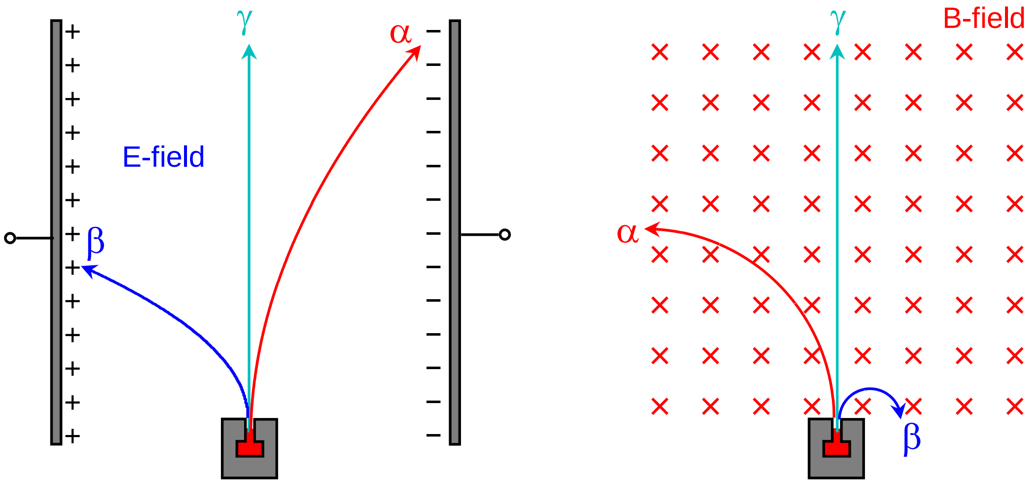
Now we know the exact composition of each radiation. Alpha and beta turn out to be energetic particles, while gamma ray is actually a very high frequency electromagnetic wave (or very energetic photons, if you prefer).

	alpha particles	beta particles	gamma ray
Symbol	${}^{4}_{2}He$	${}^{0}_{-1}e$	${}^{A}_{Z}X^* \rightarrow {}^{A}_{Z}X + \gamma$
Mass	~4u	~u/1800	0
Charge	+2e	−e	0
Speed	~0.05c	~c	c

Deflection in *E* and *B* Fields

Since alpha and beta particles are oppositely charged, they are deflected in opposite directions in electric fields. For the same electric field strength, the deflection for alpha particles is smaller. This is because of their larger mass (~8000 times) despite their larger charge (4 times). On the other hand, gamma rays are uncharged, so electric fields have totally no effect on them.

[6] If you're curious why certain nuclides undergo alpha decay whereas other nuclides undergo beta decay, you can read Appendix D: Segre chart.



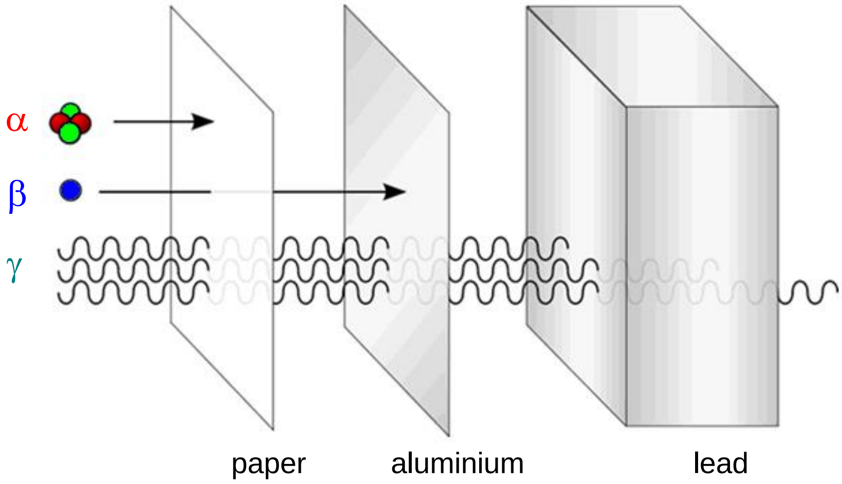
Similarly, in magnetic fields, alpha and beta particles will trace out circular paths in opposite directions. Again, the deflection for the alpha particles is much smaller thanks to their larger momentum. Gamma rays, being uncharged, do not experience any magnetic forces.

<u>Ionizing Power</u>

Alpha particles cause ionization by pulling orbital electrons out of the atoms they encounter in their track. Beta particles cause ionization by knocking out orbital electrons. It may be counter-intuitive, but the slower speed of the alpha particles is the main reason for their higher ionizing power (up to 100,000 ions per mm) compared to beta particles. The beta particles' interaction with each atom is so fleeting the rate of ionization is much lower.

Gamma rays, being uncharged, do not cause ionization directly. However, because of the large amount of energy packed in each gamma photon, they can knock out electrons from atoms they encounter at very high speed, effectively turning those electrons into beta particles. These secondary beta particles can go on to produce indirect ionization. Nevertheless, this mechanism occurs at very low rate, making gamma rays only weakly ionizing.

Penetrating Power



It may be counter-intuitive, but the higher ionizing power of the alpha particles is the reason for their shorter range compared to beta particles. Every time an alpha particle produces an ion, it loses a fraction of its kinetic energy. Since it produces so many ions per unit distance, it loses all its energy after a short distance (upon which it will absorb two electrons and become a neutral helium atom).

18.4.5 Activity

A sample of radioactive nuclei will decay over time, emitting radiation in the process.

The number of decay per unit time is known as the **activity**, *A*. It has the SI unit of Becquerel (Bq), which corresponds to 1 decay per second.

Now there is a distinction between radioactive decay and other nuclear reactions such as fusion and fission: the decay of a nucleus is (1) spontaneous and (2) random.

What is meant by spontaneous is that the decay is not triggered by anything. Nobody did nothing to that nucleus. It just happens when it happens. As such, the activity of a radioactive source is not affected by external environmental factors such as temperature, pressure, etc. There is absolutely nothing we can do to quicken or slow down the decay rate.

What is meant by random is that the decay is a random event, and the probability of decay of every nucleus in the sample is uniform. So even though nobody in this world can predict when and which nucleus is going to decay, the rate of decay for a large population is predictable.

The probability per unit time that a nucleus will decay is called the **decay constant** λ. For example, the alpha decay of radon-222 into polonium-218 has a decay constant of $2.09\times10^{-6}\ \text{s}^{-1}$. The same constant can be expressed as $1.25\times10^{-4}\ \text{min}^{-1}$, $7.52\times10^{-3}\ \text{hr}^{-1}$, 0.181 day^{-1} or even 65.9 yr^{-1}. Now we are just playing with the units. We are not saying the nucleus is going to decay 65.9 times a year. I am trying to highlight the fact that the decay constant is similar to but not exactly the same as a probability.

The activity is related to λ through the equation

$$A = \lambda N$$

where *N* is the number of (undecayed) nuclei.

For example, if we start out with 1 billion radon-222 nuclei, then we expect about $(2.09\times10^{-6})(1\times10^{9}) = 2000$ decays per second. However, 1 week later, you cannot expect the activity to be still 2000 Bq! The reason is because we no longer have 1 billion undecayed radon-222. As time progresses, the activity should drop.

If we use N_0 and N to denote the initial and current number of radioactive nuclei, and let ΔN be the change in the number over a time duration Δt , then we can write

$$\Delta N = -A\Delta t$$
$$\Delta N = -\lambda N \Delta t$$
$$\frac{1}{N}\Delta N = -\lambda \Delta t$$

Integrating both sides,

$$\int_{N_0}^{N} \frac{1}{N} dN = -\int_0^t \lambda dt$$
$$\ln \frac{N}{N_0} = -\lambda t$$
$$N = N_0 e^{-\lambda t}$$

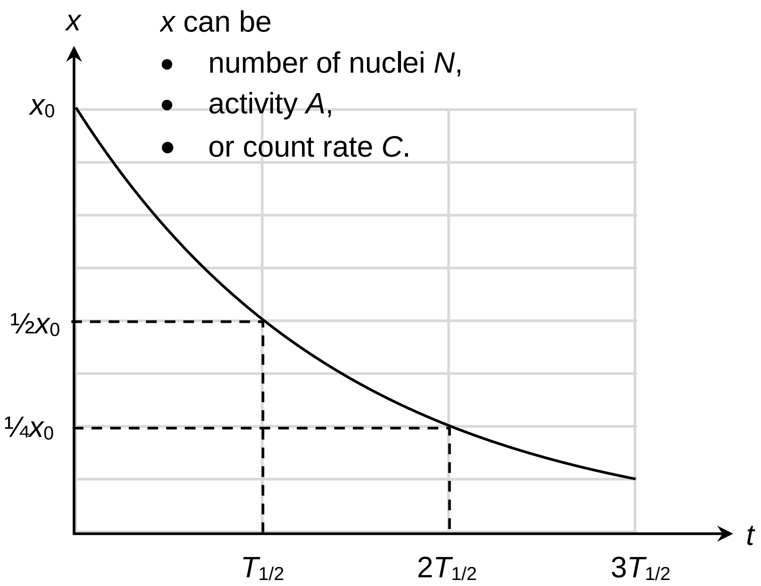
So N is going to decrease exponentially over time.

Since the activity A is proportional to the number of undecayed nuclei ($A = \lambda N$ and $A_0 = \lambda N_0$), A should also decrease exponentially over time.

$$A = A_0 e^{-\lambda t}$$

In practice, the activity is measured using a Geiger-Muller counter aka a GM counter (See Appendix E for a brief write up). Logically, the measured count rate C (due to a particular radioactive source) should be proportional to the activity. This means that C should also decrease exponentially.

$$C = C_0 e^{-\lambda t}$$



Besides the decay constant, the **half life**, $T_{1/2}$, is the other parameter that is used to characterize the rate of decay of a nuclide. It is the time it takes for about half of the nuclei in the population to decay. You can also think of it as the (average) time taken for a sample's activity to be halved.

$$0.5 = e^{-\lambda T_{1/2}}$$
$$\ln 0.5 = -\lambda T_{1/2}$$
$$T_{1/2} = \frac{\ln 2}{\lambda}$$

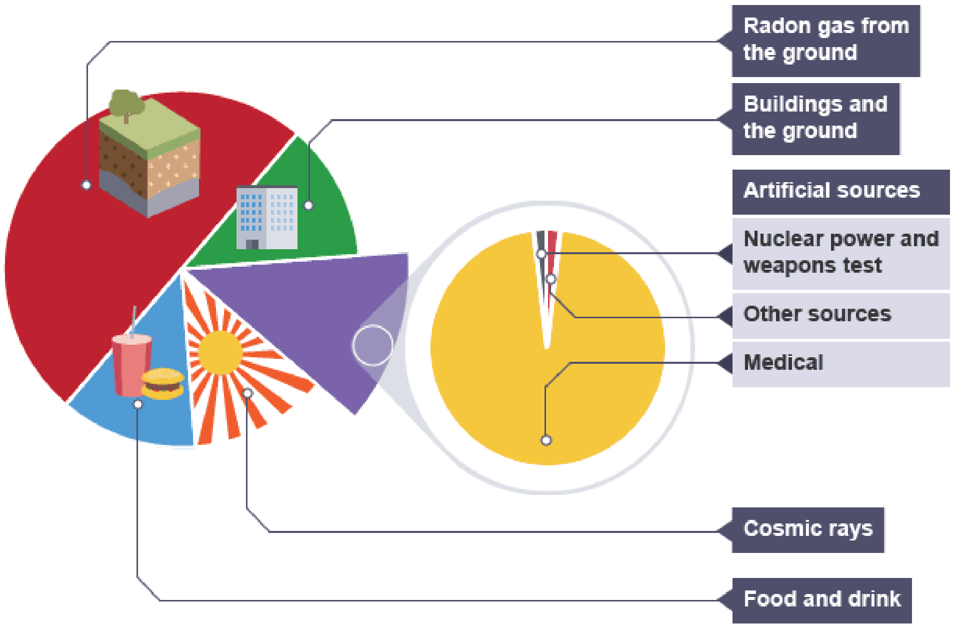
For our Rn-222 example, the half life is $T_{1/2} = \dfrac{\ln 2}{2.09 \times 10^{-9}} = 3.31 \times 10^{9}\ \text{s} = 3.8\ \text{days}$. Meaning its activity will drop from the initial 2000 Bq to 1000 Bq after 3.8 days, 500 Bq after 7.6 days, 250 Bq after 11.4 days and so on. Likewise, the fraction of undecayed nuclei in the sample is 1/2 after 3.8 days, 1/4 after 7.6 days, 1/8 after 11.4 days and so on.

Before we leave this section, let's remind ourselves that these formulae are accurate only if N is large. Radioactive decay is at its core a random event. In practice, there will always be random fluctuations in the measured count rate that result in deviations from the theoretical values.

18.4.6 Background Radiation

A GM counter detects activity even before we deliberately introduce any radioactive source near it. This is due to the background radiation that is all around us. There is practically nothing we can do to block out the background radioactive sources because they are in the air, in the soil, in the buildings, and in ourselves. What we must do when using the GM counter, is to deduct the (average) background count rate from the measured count rate to obtain the actual count rate due to the radioactive source of our interest.

Anyway, the pie chart below illustrates the major sources of background radiation we encounter in our lives.



18.5 Biological Effects of Radiation

When ionizing radiations pass through a biological environment, they cause ionization of the atoms and molecules near their trajectory. The effect of ionization on biological cells can be direct or indirect.

Direct Effect

If the radiation interacts with the atoms of the DNA molecule, or some other critical cellular components, it is referred to as a direct effect. If enough atoms are affected, the cells may die, mutate or stop replicating.

Indirect Effect

It is more likely that the radiation interacts with the water molecules in the body (which make up most of the cell volume) rather than the DNA molecules or some other critical cellular components directly. When radiation interacts with water, it may produce free radicals such as hydrogen (H) and hydroxyls (OH), which may then go on to attack the DNA molecules or some other critical cellular components.

Appendix A: Einstein's Equations

A number of concepts included in the H2 syllabus are closely related to, or "discovered" through Einstein's special theory of relativity. While the H2 syllabus only requires students to apply these concepts, I guess some students may be interested in their derivation as well.

But we'll still skip all the relativity stuff, and start from

$$K = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2 \quad \cdots\cdots\cdots(1)$$

This is the equation for the kinetic energy K of a particle, derived from work-energy theorem (using relativistic force $F = \frac{ma}{\sqrt{1 - v^2/c^2}}$). [7]

If you do a binomial expansion, equation (1) will collapse to become the familiar $K = \frac{1}{2}mv^2$ for $v << c$. Hopefully this makes you more accepting of this equation.

Anyway, Einstein interpreted the first term in the equation (1) to represent the total energy E of the particle, and the second term to be energy the particle has even when it is at rest (so that K is the energy due to motion only). Re-arranging the equation, we have

$$K + mc^2 = E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \quad \cdots\cdots\cdots(2)$$

The energy mc^2 comes to be known as the rest energy. It is from here that we get all that mass-equivalence stuff.

But this is not all that Einstein milked from this equation.

First, he wrote equation (2) as

$$\frac{E}{mc^2} = \frac{1}{\sqrt{1 - v^2/c^2}} \quad \cdots\cdots\cdots(3).$$

Then, from relativistic momentum $p = \frac{mv}{\sqrt{1 - v^2/c^2}}$, he wrote

[7] You'll see the Lorentz factor $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$ all over the place. Don't be too intimidated. Just take it as the factor by which stuff gets "distorted" when particles travel at speed close to c.

$$\frac{p}{mc} = \frac{v/c}{\sqrt{1-v^2/c^2}} \quad \cdots\cdots\cdots(4)$$

Then by doing $(3)^2 - (4)^2$, he got rid of v in the equations and arrived at

$$E^2 = (pc)^2 + (mc^2)^2 \quad \cdots\cdots\cdots(5)$$

For a particle at rest ($p = 0$), the equation becomes

$$E = mc^2$$

So $E = mc^2$, the most famous physics equation, is actually a special case of the more general equation (5).

For the other special case of massless particles ($m = 0$), the equation becomes

$$E = pc$$

This suggests that a massless particle may have momentum despite not having any mass!

For photons which has energy $E = \frac{hc}{\lambda}$, we obtain

$$\frac{hc}{\lambda} = pc \;\Rightarrow\; p = \frac{h}{\lambda}$$

This is where we obtain the formula for the momentum of a photon.

All de Broglie did was to assume that $p = \frac{h}{\lambda}$ is equally valid for even massful particles travelling at non c speed. Voilah, de Broglie got his namesake formula (and a Nobel Prize).

$$\lambda = \frac{h}{p}$$

Appendix B: Nuclear Force

The stability (or instability) of a nucleus is determined by the competition between

1. the attractive nuclear force among the protons and neutrons and
2. the repulsive electrical force among the protons.

What are the characteristics of the nuclear force?

1. It does not depend on charge: the binding for neutron-neutron, proton-proton and neutron-proton are exactly the same.
2. It has an extremely short range of about one nucleon, i.e. 10^{-15} m. Within this range, the nuclear force is much stronger than electrical forces. (It has got to be, considering how closely the protons are being held together). Beyond this range, the nuclear force abruptly drops to zero.

We can now explain why the most stable nuclides are not too big, not too small, with about 50 nucleons. Electrical repulsion are long range forces compared to nuclear forces. While a proton repels every other proton in the nucleus, even if they are at opposite ends of the nucleus, a nucleon can only attract its immediate neighbours.

When the nucleus is small, adding another nucleon usually results in a more stable nucleus because of the attractive nuclear forces it brings. Beyond a certain size, adding another proton actually results in a less stable nucleus because the longer range electrical repulsion starts to dominate over the shorter range nuclear forces.

Why not we just keep adding neutrons? Since they contribute to the binding force without introducing any electrical repulsion? Unfortunately, neutrons themselves have a tendency to decay into a proton and an electron. In fact, it turns out the presence of protons is necessary to help keep the neutrons stable. This explains why there is a limit to how big a nucleus can be formed. In fact, lead-208 is the largest stable nuclide.

Appendix C: Magic Numbers

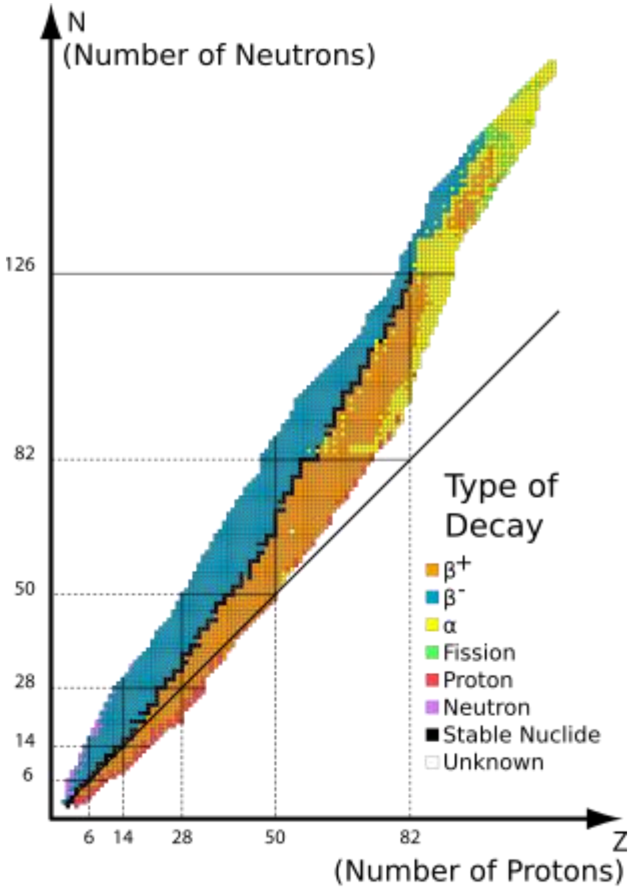
In chemistry, atoms with filled outer shells are most unreactive, making the atoms with atomic number $Z = 2, 10, 18, 36, 54, 86$ (the noble gases) the most stable elements. It turns out that in the nucleus, protons and neutrons are also arranged in shells and subshells, just that the numbers are different. When the number of the protons (Z) or the number of neutrons (*N*) is 2, 8, 20, 28, 50, 82 or 126, the resulting nucleus tends to be more stable. This "explains" why doubly magic nuclides (nuclides whose *Z* and *N* are both magic) such as

$$ {}^{4}_{2}He \quad {}^{16}_{8}O \quad {}^{40}_{20}Ca \quad {}^{48}_{20}Ca \quad {}^{208}_{82}Pb $$

are substantially more stable compared to their neighboring nuclei of similar sizes.

Appendix D: Segre chart

Out of about 2500 known nuclides, less than 300 (shown in black) are stable. When plotted on a Segre chart (see below), the stable nuclides are seen to line up along what's called the valley of stability. Apparently, the *N*/*Z* ratio for stable nuclei increases as the nuclei grow in size.

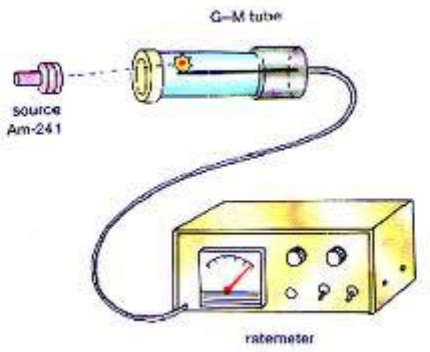


Some insights we can draw from the Segre chart are:

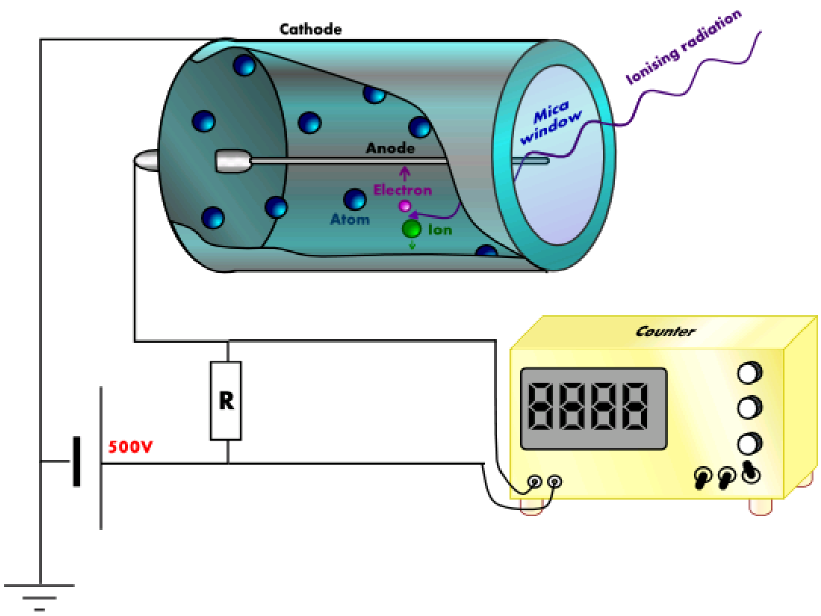
- Lead-208 is the largest known stable nuclide. Nuclei larger than lead-208 tend to undergo alpha decay (shown in yellow) to quickly shed some "unwanted" nucleons.
- The nuclei above the valley of stability have too many neutrons and too few protons. As such, they tend to undergo beta-minus decay (shown in blue) to swap a neutron for a proton (and an electron).
- The nuclei below the valley of stability have too many protons and too few neutrons. As such, they tend to undergo beta-plus decay (shown in orange) to swap a proton for a neutron (and a positron).

Appendix E: Geiger-Muller Counter

A Geiger-Muller counter consists of a tube (aka Geiger–Muller tube) and a ratemeter (aka scaler). The GM Tube is the sensing element. The ratemeter contains the electronics to do the counting and display the reading in count per minute (cpm) or other units.



A Geiger–Müller tube is filled with a low-pressure (~0.1 atm) inert gas. At one end, it has a **very thin** mica-window through which radiation can enter. The walls of the tube form the cathode. The anode is a wire passing up the center of the tube. A potential difference of **several hundred volts** is maintained between the cathode and anode.



When an ionizing radiation makes its way through the mica-window into the tube, it may ionize one or a few gas molecules. The resulting charged particles are immediately accelerated by the strong electric field in the tube, and may pick up enough KE so they can in turn ionize more atoms through collisions. If all goes well, a chain reaction will culminate in an avalanche breakdown, which translate into a short, intense pulse of current which passes between the electrodes. These pulses are measured or counted by the ratemeter. The ratemeter usually also includes a loudspeaker which gives a “click” for each pulse. This gives rise to the cute clicking sound you hear.

A few things worth noting:

- The count rate is not equal to the activity of the radioactive source. Firstly, not all the radiations enter the tube. Secondly, not every radiation produce one pulse. Gamma radiation in particular is only weakly ionising so only a tiny fraction of gamma photons “succeed” in producing a pulse. So a low activity beta source can produce more clicks per minute than a high activity gamma source.
- A pulse is a pulse, whether caused by high or low energy radiation. So the count rate is not a measure of the energy of radiation. In fact, the more energetic the gamma radiation, the more likely it will pass right through the tube. So a high energy gamma source can produce a lower count rate than a low energy one.
- A pulse is a pulse, whether caused by an alpha particle, beta particle or a gamma photon. A simple trick employed to identify the type of radiation is to insert a suitable absorber between the radiation source and the GM tube. For example, if the count rate is reduced drastically by a piece of paper, we have an alpha source.
- Even in the absence of any radiation source, GM counters produce a background pulse rate, due to cosmic rays and other background radiation. This background count rate must be subtracted from the measured count rate to obtain count rate due to the radioactive source alone.